Review Day one Solutions.

#1. Concept: $f'(x) = 1 \Rightarrow$ solve for x.

Work: $f'(x) = \dfrac{(1)\sqrt{x^2+1} - (\frac{1}{\cancel{2}})(\cancel{2}x^2)(x^2+1)^{-\frac{1}{2}}}{x^2+1} = 1$ (Apply Quotient Rule and Chain Rule)

$\Rightarrow \dfrac{(x^2+1)^{\frac{1}{2}} - x^2(x^2+1)^{-\frac{1}{2}}}{x^2+1} = 1$ (Multiply (x^2+1) both sides)

$\Rightarrow (x^2+1)^{\frac{1}{2}} - x^2(x^2+1)^{-\frac{1}{2}} = x^2+1$ (Multiply $(x^2+1)^{\frac{1}{2}}$ both sides)

$\Rightarrow \cancel{x^2}+1 - \cancel{x^2} = (x^2+1)(x^2+1)^{\frac{1}{2}}$

$\Rightarrow (1)^{\frac{2}{3}}\left[(x^2+1)^{\frac{3}{2}}\right]^{\frac{2}{3}} \Rightarrow 1 = x^2+1 \Rightarrow x^2 = 1 \Rightarrow \boxed{x=0}$

$\therefore$ where $x=0$, $f'(x) = 1$.

#2. (a) Apply Chain Rule and Quotient Rule. $\Rightarrow$ Then Simplify.

$y' = \left[\dfrac{x^{\cancel{2}}}{x+3}\right]\left[\dfrac{(1)x^2 - 2x(x+3)}{\cancel{x^4}\ x^2}\right]$

$= \dfrac{x^2 - 2x^2 - 6x}{x^2(x+3)}$

$= \dfrac{-x^2-6x}{x^2(x+3)} = \dfrac{-\cancel{x}(x+6)}{x^{\cancel{2}}(x+3)} = \boxed{\dfrac{-(x+6)}{x(x+3)}}$

(b) Apply product Rule and Chain Rule

$y' = \cos x \cdot \cos^3(2x^2+5) + \sin x \cdot 3\cos^2(2x^2+5)[-\sin(2x^2+5)](4x$

$y' = \boxed{\cos x \cdot \cos^3(2x^2+5) - 12x \cdot \sin x \cos^2(2x^2+5)\sin(2x^2+5)}$

Find $f'(x)$ and Simplify $\Rightarrow$ then find $f''(x)$

#3 $f(x) = \sqrt{x} \cdot \cos(4x)$

$$f'(x) = \frac{1}{2}(X)^{-\frac{1}{2}} \cos(4x) + \sqrt{X}(4)(-\sin 4x)$$

$$= \frac{\cos 4x}{2\sqrt{X}} + \frac{-4X \sin 4x}{\sqrt{x}}$$

Method one.

$$= \frac{\cos 4x - 8x \sin 4x}{2\sqrt{x}}$$

$$f''(x) = \frac{(-4\sin 4x - 8\sin 4X - 32X\cos 4x)2\sqrt{X} - \cancel{2}\cdot\frac{1}{\cancel{2}}(x)^{-\frac{1}{2}}[\cos 4x - 8x\sin 4x]}{4X}$$

$$= \frac{(-12\sin 4x - 32\cos 4x)2\sqrt{X} - (X)^{-\frac{1}{2}}(\cos 4x - 8X\sin 4x) \cdot \sqrt{x}}{4x \cdot \sqrt{x}}$$

$$= \frac{(-12\sin 4x - 32\cos 4x)\cdot 2X - (\cos 4x - 8X\sin 4x)}{4X\cdot\sqrt{X}}$$

$$= \frac{-24X\sin 4X - 64X\cos 4X - \cos 4X + 8X\sin 4X}{4X\sqrt{X}}$$

$$= \frac{-16X\sin 4X - 64X\cos 4X - \cos 4X}{4X\sqrt{X}}$$

Method two

$$f'(x) = \frac{1}{2}(X)^{-\frac{1}{2}}\cos 4x + 4\sqrt{X}(-\sin 4x)$$

$$f''(x) = -\frac{1}{4}(X)^{\frac{3}{2}}\cos 4X + \cancel{\frac{1}{2}}(X)^{-\frac{1}{2}}\cdot \overset{2}{\cancel{4}}(-\sin 4x) - 2X^{-\frac{1}{2}}\sin 4x - 16\sqrt{X}\cos 4X$$

$$= \frac{-\cos 4X}{4X\sqrt{X}} - \frac{4\sin 4X}{\sqrt{X}} - \frac{2\sin 4X}{\sqrt{X}} - 16\sqrt{x}\cos 4X$$

$$= \frac{-\cos 4x}{4x\sqrt{x}} - \frac{4\sin 4x}{\sqrt{X}} - 16\sqrt{X}\cos 4X = \frac{-\cos 4x - 16x\sin 4x \overset{-64x\cos 4x}{\downarrow}}{4X\sqrt{X}}$$

#4. $y = 3\sin bx - a\cos 2x$

$$\frac{dy}{dx} = 3b\cos bx + 2a\sin 2x$$

$$\frac{d^2y}{dx^2} = -3b^2\sin bx + 4a\cos 2x$$

$$\Rightarrow \quad y + \frac{d^2y}{dx^2} = 6\cos 2x$$

$$\Rightarrow \boxed{3\sin bx} - a\cos 2x \; \boxed{+(-3b^2\sin bx)} + 4a\cos 2x = 6\cos 2x$$

$\Rightarrow 3\sin bx - 3b^2\sin bx = 0$

$\Rightarrow 3 - 3b^2 = 0$

$\Rightarrow b = \pm 1$

$\Rightarrow 4a\cos 2x - a\cos 2x = 6\cos 2x$

$4a - a = 6$

$3a = 6$

$\boxed{a = 2}$