21B Antiderivatives

1. Given $\frac{dy}{dx} = x^2$, find y in terms of x.

$$\boxed{y = \frac{1}{3}x^3 + C}$$

If $F'(x) = f(x)$, then

- $f(x)$ is the derivative of $F(x)$.
- $F(x)$ is an antiderivative of $f(x)$.

Indefinite Integral

If $F'(x) = f(x)$, then $F(x)$ is an antiderivative of $f(x)$.

$\int \frac{dy}{\cancel{dx}} \cancel{dx} = y$.

Rules for Integration (antiderivative)-you can find these in the IB formula booklet.

$\int k\,dx = \boxed{kx + C}$	$\int \sin x\,dx = \boxed{-\cos x + C}$
$\int x^n\,dx = \boxed{\frac{x^{n+1}}{n+1} + C}$	$\int \cos x\,dx = \boxed{\sin x + C}$
$\int e^x\,dx = \boxed{e^x + C}$	$\int \sec^2 x\,dx = \boxed{\tan x + C}$
$\int \frac{1}{x}\,dx = \boxed{\ln x + C}$	$\int \sec x \tan x\,dx = \boxed{\sec x + C}$

- $\frac{d}{dx}(3^x) = (\ln 3) \cdot 3^x$
- $\int 3^x\,dx = \frac{3^x}{\ln 3} + C$

2. Find an antiderivative of e^{5x}.

$$\int e^{5x}\,dx = \boxed{\frac{1}{5}e^{5x} + C}$$

3. If $y = \sin 3x$, find $\frac{dy}{dx}$. Hence find $\int \cos 3x\,dx$.

$$\frac{dy}{dx} = 3\cos 3x$$

$$\boxed{\frac{1}{3}\sin 3x + C}$$

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a. If $\frac{dy}{dx} = x^4 + 7$, find y

b. Evaluate $\int (x^4 + 7)\,dx$

c. Find $f(x)$ if $f'(x) = x^4 + 7$

$$\int (x^4 + 7)\,dx = \boxed{\frac{1}{5}x^5 + 7x + C}$$

$$\int f(x)\,dx = F(x) + C$$

Indefinite Integral of $f(x)$.

- $\int e^{kx}\,dx = \frac{1}{k} \cdot e^{kx} + C$.
- $\int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + C$

Differentiate each function with respect to x.

1. $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x + 7$

$\frac{dy}{dx} = -2x^3 + 5x^{\frac{2}{3}} + 2$

2. $y = \frac{3}{x^3} - \sqrt[4]{x^9} = 3x^{-3} - x^{\frac{9}{4}}$

$y' = -9x^{-4} - \frac{9}{4}x^{\frac{5}{4}}$

3. $y = (5x^2+4)^6$

$y' = 6(5x^2+4)^5(10x)$

4. $y = \sqrt[3]{8-x^5} = (8-x^5)^{\frac{1}{3}}$

$y' = \frac{1}{3}(8-x^5)^{-\frac{2}{3}}(-5x^4) = \frac{-5x^4}{3}(8-x^5)^{-\frac{2}{3}}$

5. $y = \sin(2x^3)$

$y' = 6x^2\cos(2x^3)$

6. $y = \sqrt{x}\tan x$

$y' = \frac{1}{2}x^{-\frac{1}{2}}\tan x + \sqrt{x}\sec^2 x$

$= \frac{\tan x}{2\sqrt{x}} + \sqrt{x}\sec^2 x$

Given $f'(x)$, find $f(x)$

1. $f'(x) = 3x^2$

$f(x) = x^3 + C$

2. $f'(x) = 2x - 4x^4$

$f(x) = x^2 - \frac{4}{5}x^5 + C$

3. $f'(x) = 2x^2 + \sqrt{x} = 2x^2 + x^{\frac{1}{2}}$

$f(x) = \frac{2}{3}x^3 + \frac{2}{3}x^{\frac{3}{2}} + C$

4. $f'(x) = \frac{2}{3}x^2 - \cos x$

$f(x) = \frac{2}{9}x^3 - \sin x + C$

5. $f'(x) = 5x^4 - \sqrt[3]{x} = 5x^4 - x^{\frac{1}{3}+\frac{3}{3}}$

$f(x) = x^5 - \frac{3}{4}x^{\frac{4}{3}} + C$

6. $f'(x) = 2\sin 5x$

$f(x) = -\frac{2}{5}\cos(5x) + C$

7. $f'(x) = \frac{1}{x} - e^x$

$f(x) = \ln x - e^x + C$

8. $f'(x) = \frac{1}{2}\sqrt{x} - \sin 2x = \frac{1}{2}x^{\frac{1}{2}} - \sin 2x$

$f(x) = \left(\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{3}\right)x^{\frac{3}{2}} + \frac{1}{2}\cos 2x + C$

$= \frac{1}{3}x^{\frac{3}{2}} + \frac{1}{2}\cos 2x + C$

9. $f'(x) = e^{4x} + 5^x$

$f(x) = \frac{1}{4}e^{4x} + \frac{1}{\ln 5}\cdot 5^x + C$

10. $f'(x) = \sec x\tan x - \cancel{\sec^2 x}$

$f(x) = \sec x - \tan x + C$