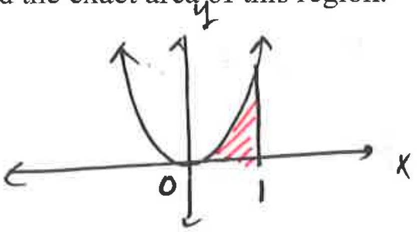
22A Area under between two Curves Day one

Warm Up

Solution:

Sketch the region bounded by $y = x^2$, the x-axis, and $x = 1$.
Find the exact area of this region.



$$\Rightarrow \int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_{x=0}^{x=1} = \boxed{\frac{1}{3}}$$

Area between two curves.

$$h = f(x) - g(x)$$

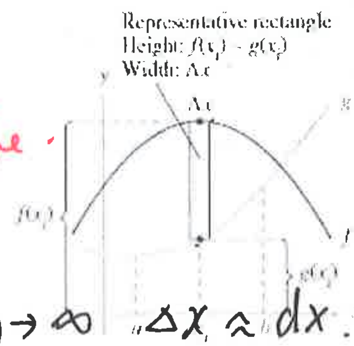
upper curve

lower curve

$$\lim_{n\to\infty}\sum_{i=1}^{n}[f(x_i) - g(x_i)]\Delta x = \int_a^b [f(x) - g(x)]\,dx \quad \text{(FTC)}$$

$$\Delta x = \frac{b-a}{n}$$

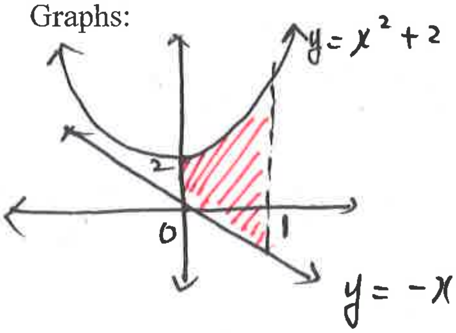
$$\Delta x \to 0 \Rightarrow n \to \infty, \quad \Delta x \approx dx.$$



Example 1) Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$. $x = 0$, and $x = 1$.

Solution:

Graphs:



$$\Rightarrow \int_0^1 [(x^2+2) - (-x)]\,dx = \int_0^1 (x^2 + 2 + x)\,dx$$

$$= \left[\frac{1}{3}x^3 + 2x + \frac{1}{2}x^2\right]_0^1$$

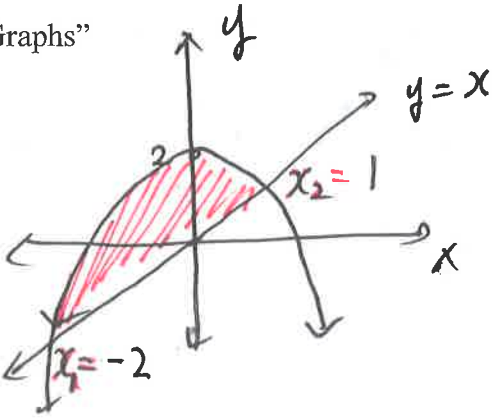
$$= \frac{1}{3} + 2 + \frac{1}{2} = \boxed{\frac{17}{6}}$$

$\frac{12}{5}$

Example 2) Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.

Solution:

Graphs"



$2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

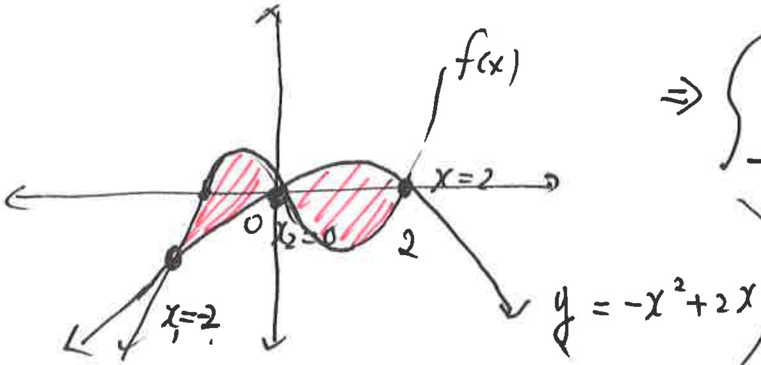
$x = -2, \; x = 1$

$$\Rightarrow \int_{-2}^{1} [(2 - x^2) - x]\,dx$$

$$= \left[2x - \frac{1}{3}x^3 - \frac{1}{2}x^2\right]_{x=-2}^{x=1}$$

$$= \left[2 - \frac{1}{3} - \frac{1}{2}\right] - \left[-4 + \frac{8}{3} - 2\right]$$

$$= 8 + \left(-3 - \frac{1}{2}\right) = \boxed{\frac{9}{2}}$$



$$\Rightarrow \int_{-2}^{0} (f(x) - g(x))\,dx + \int_{0}^{2} (g(x) - f(x))\,dx.$$

$$3x^3 - \cancel{x^2} - 10x = -\cancel{x^2} + 2x$$

$$3x^3 - 12x = 0$$

$$3x(x^2 - 4) = 0$$

$$3x(x+2)(x-2) = 0$$

$$x = 0,\ x = -2,\ x = 2$$

$$\int_{-2}^{0} \left[(3x^3 - \cancel{x^2} - 10x) - (-\cancel{x^2} + 2x)\right] dx$$

$$+ \int_{0}^{2} \left[(-\cancel{x^2} + 2x) - (3x^3 - \cancel{x^2} - 10x)\right] dx$$

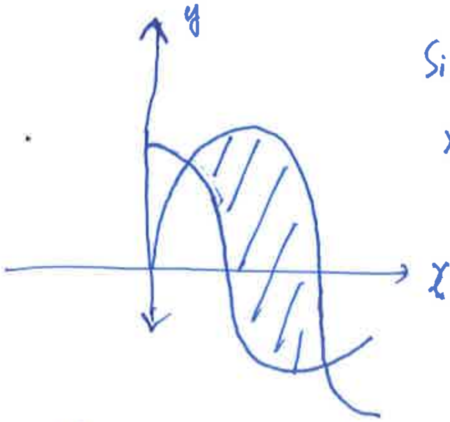
$$= \int_{-2}^{0} (3x^3 - 12x)\,dx + \int_{0}^{2} (-3x^3 + 12x)\,dx$$

$$= \left[\frac{3}{4}x^4 - 6x^2\right]_{-2}^{0} + \left[-\frac{3}{4}x^4 + 6x^2\right]_{0}^{2}$$

$$= 0 - \left[\frac{3}{4}(-2)^4 - 6(-2)^2\right] + \left[-\frac{3}{4}(2)^4 + 6(2)^2\right]$$

$$= 24 + 24 - \cancel{2}\left(\frac{3}{\cancel{4}_{2}}\right)(\cancel{16}^{8}) = \boxed{24}$$

practice)



$\sin x = \cos x$

$x = \frac{\pi}{4},\ \frac{5\pi}{4}.$

$$\Rightarrow \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} [\sin x - \cos x] = \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = \left[-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right] - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2}}$$