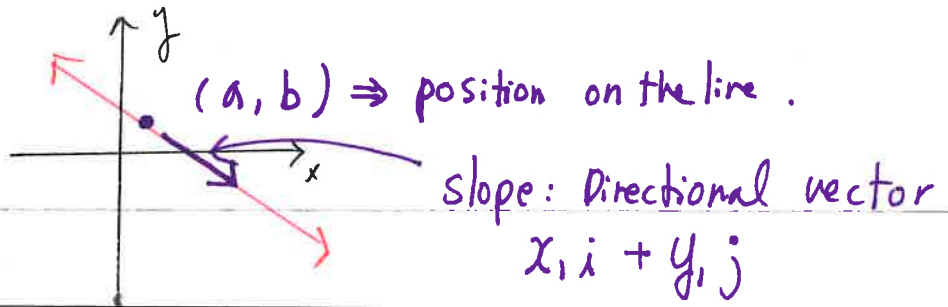


1. Lines in 2-D.



Cartesian Equation	$y = mx + b, Ax + By = C.$ $\frac{x-a}{x_1} = \frac{y-b}{y_1}$
Parametric Equation	$x = a + x_1t$ $y = b + y_1t$
Vector Equation	$\vec{r} = x_1i + y_1j \Rightarrow \vec{r} = \begin{pmatrix} a \\ b \end{pmatrix} + \lambda \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \Leftrightarrow \vec{r} = (a_1i + b_1j) + \lambda(x_1i + y_1j)$

Example 1) Given the graph, Write an equation of the line

a. In Cartesian form

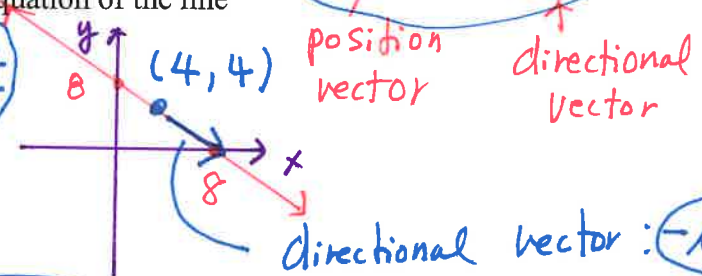
$\frac{x-4}{-1} = \frac{y-4}{1}$

b. Parametric form

$x = 4 - t$
 $y = 4 + t$

c. Vector form.

$\vec{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \end{pmatrix}$



directional vector: $-i + j$ ✓
OR $i - j$ ✓

Example 2) Given $y = 3x - 5$

a. Write an equation in the form of $\frac{x-a}{x_1} = \frac{y-b}{y_1}$

$(1, -2)$ OR $(0, -5)$

b. Write an equation in a parametric form.

$x = 1 + t$
 $y = -2 + 3t$

c. Write an equation in vector form.

$\vec{r} = (i - 2j) + \lambda(i + 3j)$

directional vector: $i + 3j$
OR $-i - 3j$

$\frac{x-1}{1} = \frac{y+2}{3}$

$x = \frac{y+5}{3}$

directional vector $\Rightarrow \begin{pmatrix} -2-1 \\ 5-3 \end{pmatrix} = -3i + 2j$

Example 3) A line is passing through $(-2, 5)$ and $(1, 3)$.

a. Write an equation in the form of $\frac{x-a}{x_1} = \frac{y-b}{y_1}$

$\frac{x+2}{-3} = \frac{y-5}{2}$

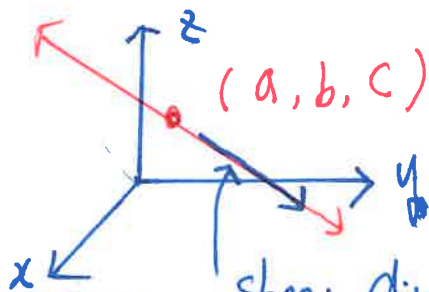
b. Write an equation in a parametric form.

$x = -2 - 3t$
 $y = 5 + 2t$

c. Write an equation in vector form.

$\vec{r} = (-2i + 5j) + \lambda(-3i + 2j)$

2. Lines in 3-D.



Cartesian Equation	$\frac{x-a}{x_1} = \frac{y-b}{y_1} = \frac{z-c}{z_1} = \underline{x_1 i + y_1 j + z_1 k}$
Parametric Equation	$\begin{cases} x = a + x_1 t \\ y = b + y_1 t \\ z = c + z_1 t \end{cases}$ Directional Vector
Vector Equation	$\vec{r} = x_1 i + y_1 j + z_1 k \Rightarrow \vec{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + t \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ ↑ position vector

Example 1) Given the line is passing through (2, -1, 4) and (-1, 0, 2).

a. Write an equation in parametric form.

$$x = 2 + 3t, \quad y = -1 - t,$$

b. Write an equation in vector form.

$$z = 4 + 2t$$

$$\vec{d} = \begin{pmatrix} 2+1 \\ -1-0 \\ 4-2 \end{pmatrix} = 3i - j + 2k$$

c. Write an equation in Cartesian form.

$$\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-4}{2}$$

$$\vec{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

Example 2) Given the line is passing through (4, 2, 5) with directional vector $i - j + 2k$

a. Write an equation in parametric form.

$$\begin{cases} x = 4 + t \\ y = 2 - t \\ z = 5 + 2t \end{cases}$$

b. Write an equation in vector form.

$$\vec{r} = (4i + 2j + 5k) + t(i - j + 2k)$$

c. Write an equation in Cartesian form.

$$\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-5}{2}$$

Example 3) Given the line $\frac{2-x}{4} = \frac{y+1}{2} = \frac{4-z}{6}$

a. Write an equation in parametric form.

$$\Rightarrow \frac{x-2}{-4} = \frac{y+1}{2} = \frac{z-4}{-6}$$

b. Write an equation in vector form.

$$\vec{r} = (2i - j + 4k) + t(-4i + 2j - 6k)$$

$$\begin{cases} x = 2 - 4t \\ y = -1 + 2t \\ z = 4 - 6t \end{cases}$$