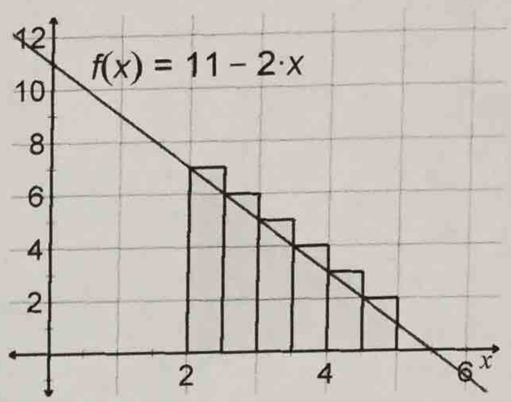
IB Math HL 1 21A Area under a Curve Name________________________

<u>Goal</u>: Estimate the area under a curve using rectangle approximations.

1. The graph of $f(x)=11-2x$ is shown. We are going to approximate the area under this function on the interval $[2,5]$.



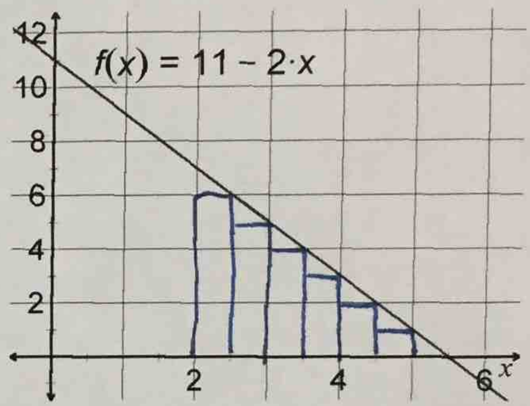
a. Six rectangles have been drawn. What is the width of each rectangle?

$\frac{1}{2}$

b. These rectangles are called left-hand rectangles because the height of each rectangle is determined by the function's height at the left x-value of the interval. For example, the height of the rectangle at $x=2$ is 7. Approximate the area under the curve by finding the sum of the areas of all six rectangles.

$\frac{1}{2}\cdot 7+\frac{1}{2}\cdot 6+\frac{1}{2}\cdot 5+\frac{1}{2}\cdot 4+\frac{1}{2}\cdot 3+\frac{1}{2}\cdot 2$

13.5



c. Is the approximation from part b too high or too low? How can you tell?

too high - there are little pieces of rectangle above the line

d. Sketch 6 right-hand rectangles and compute this approximate area.

$\frac{1}{2}\cdot 6+\frac{1}{2}\cdot 5+\frac{1}{2}\cdot 4+\frac{1}{2}\cdot 3+\frac{1}{2}\cdot 2+\frac{1}{2}\cdot 1$

10.5

2. Let's approximate the area under the curve using 30 left-hand rectangles!

a. How wide will each rectangle be? $\frac{1}{10}$

b. If we are using 30 left-rectangles, explain why the area of the first rectangle can be written as $\frac{1}{10}f(2)$.

width height of left rectangle

c. Explain why the total area of all 30 left-rectangles can be written as

30th rectangle width height

$$A=\sum_{k=0}^{29}0.1f(2+0.1k)$$

move interval by .1 each time

start at 2 (left end of 1st interval)

d. Use your graphing calculator to evaluate the sigma expression in part c. 12.3

TI-84
Enter the function using Y =
QUIT to home screen
LIST, MATH, 5:SUM
LIST, OPS, 5:SEQ
Make it look like this:
sum(seq(.1Y_1(2+.1K), K, 0, 29, 1))

(To get Y_1, VARS, Y-VARS, 1, 1)

TI-Nspire
Enter the function in Graphs
Add a new Calculator page
Menu, 4:Calculus, 3:Sum
Make it look like the sigma notation

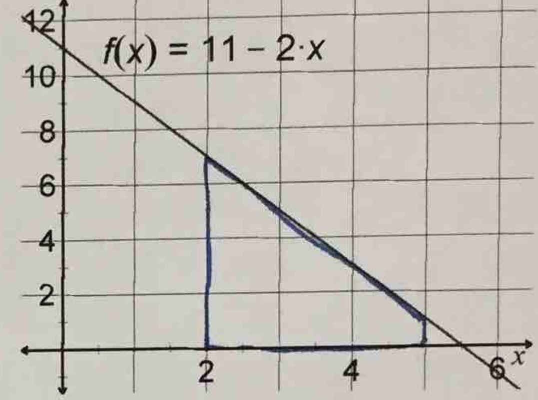
3. Now let's approximate the area under the curve using 100 right-hand rectangles!

a. Complete the sigma notation: $A = \sum_{k=1}^{100} .03 \cdot f(2 + .03k)$

b. Use your graphing calculator to evaluate the sigma expression in part b. 11.91

c. Compare this result to that of #2. Which is more accurate? Why?

11.91 - the areas above the line are smaller



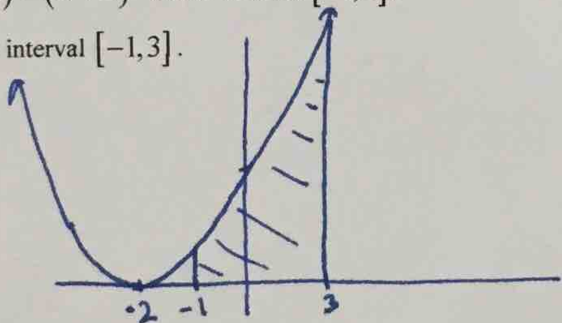
4. a. Use your results to estimate the actual area under $f(x) = 11 - 2x$ on the interval $[2,5]$.

b. Use your knowledge of geometry to calculate the actual area.

$A_{trap} = \frac{b_1 + b_2}{2} \cdot h = \frac{1+7}{2} \cdot 3 = \boxed{12}$

5. Approximate the area under $g(x) = (x+2)^2$ on the interval $[-1,3]$.

a. Sketch $g(x) = (x+2)^2$ on the interval $[-1,3]$.



b. Will left or right rectangles give a lower estimate of the area?

left

c. Using the indicated number of rectangles, write in the sigma notation and approximate the upper and lower areas.

Number of rectangles	Lower Sigma Notation	Lower Area	Upper Sigma Notation	Upper Area
8	$\sum_{k=0}^{7} \frac{4}{8} g(-1 + \frac{4}{8} \cdot k)$	35.5	$\sum_{k=1}^{8} \frac{4}{8} g(-1 + \frac{4}{8}k)$	47.5
50	$\sum_{k=0}^{49} \frac{4}{50} g(-1 + \frac{4}{50}k)$	40.3776	$\sum_{k=1}^{50} \frac{4}{50} g(-1 + \frac{4}{50}k)$	42.2976
1000	$\sum_{k=0}^{999} \frac{4}{1000} g(-1 + \frac{4}{1000}k)$	41.2853	$\sum_{k=1}^{1000} \frac{4}{1000} g(-1 + \frac{4}{1000}k)$	41.3813

d. Based on your results, estimate the exact area of this region.

21A.1 (1, 2, 3, 5a)

41.33