Review Day Two Solutions. ①

#1. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ (Definition of the derivative) and $f(x)=x^n$

$(x+h)^n = x^n + \binom{n}{1}x^{n-1}\cdot h + \binom{n}{2}x^{n-2}\cdot h^2 \cdots h^n$

$\Rightarrow \lim_{h\to 0} \frac{[\cancel{x^n} + \binom{n}{1}\cdot x^{n-1}\cdot h + \binom{n}{2}x^{n-2}\cdot h^2 \cdots h^n] - \cancel{x^n}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}[\binom{n}{1}\cdot x^{n-1} + \binom{n}{2}x^{n-2}\cdot h \cdots h^{n-1}]}{\cancel{h}}$

$= \lim_{h\to 0} \binom{n}{1}\cdot x^{n-1} + \cancel{\binom{n}{2}x^{n-2}\cdot h}^{\,0} \cdots \cancel{h^{n-1}}^{\,0}$ (Notes: $\binom{n}{1}=n$)

$= \boxed{n\cdot x^{n-1}}$

#2. Concept: Implicit Differentiation.

(a) $\ln(2y+1) = x\cdot e^y$

$\Rightarrow \left(\frac{1}{2y+1}\right)\cdot 2\cdot y' = e^y + x\cdot e^y\cdot y'$

$\Rightarrow \left(\frac{2}{2y+1}\right)\cdot y' - x\cdot e^y\cdot y' = e^y$

$\Rightarrow y'\left[\frac{2}{2y+1} - xe^y\right] = e^y$

$\Rightarrow y'\left[\frac{2 - x\cdot e^y(2y+1)}{2y+1}\right] = e^y$

$\Rightarrow \boxed{y' = \frac{e^y(2y+1)}{2 - x\cdot e^y(2y+1)}}$

(b) $e^x\cdot y - x\cdot y^2 = 2x$

$\Rightarrow e^x\cdot y' + e^x y - y^2 + 2xy\cdot y' = 2$

$\Rightarrow y'[e^x - 2xy] = 2 + y^2 - e^x y$

$\Rightarrow \boxed{y' = \frac{2 + y^2 - e^x y}{e^x - 2xy}}$

②

#3. When $x=0 \Rightarrow e^0 \cdot y - (0)(y^2) = 2\cdot 0$

$$y = 0.$$

$$y' = \frac{2+0-e^0 \cdot 0}{e^0 - 2\cdot(0)(0)} = 2.$$

The Equation of the tangent line at $x=c$: $y - f(c) = f'(c)(x-c)$ point-slope form.

$$\Rightarrow \boxed{y = 2x}$$

#4. $x^2 + 2x^2y + 4y^2 = 9$

$$\Rightarrow 2x + 4xy + \boxed{2x^2y'} + \boxed{8y\cdot y'} = 0$$

$$\Rightarrow y'[2x^2+8y] = -2x-4xy$$

$$\Rightarrow y' = \frac{-2x-4xy}{2x^2+8y} = \frac{-x-2xy}{x^2+4y}$$ Substitute y'

$$\Rightarrow y'' = \frac{[-1-2y-2x\cdot y'][x^2+4y] + [x+2xy](2x+4y')}{(x^2+4y)^2}$$

$$= \frac{[1-2y-2x\left(\frac{-x-2xy}{x^2+4y}\right)](x^2+4y) + (x+2xy)[2x+4\left(\frac{-x-2xy}{x^2+4y}\right)]}{(x^2+4y)^2}$$

#5. $f(x) = \dfrac{x^2}{2^x}$

$$f'(x) = \frac{2x\cdot\cancel{2^x} - \ln 2\cdot\cancel{2^x}\cdot x^2}{(2^x)^{\cancel{2}}} = \frac{2x - x^2\ln 2}{2^x}$$

Concept: Min/Max is where $f'(x)=0$

$x(2 - x\cdot\ln 2) = 0$ Since $2^x \neq 0$

$$\boxed{x=0 \quad , \quad x = \frac{2}{\ln 2}}$$

Concept: Inflection point(s) are where $f''(x)=0$

$$f'(x) = \frac{2x - x^2\ln 2}{2^x}$$

$$f''(x) = \frac{(2 - 2x\cdot\ln 2)\cdot\cancel{2^x} - \ln 2\cdot\cancel{2^x}(2x - x^2\ln 2)}{(2^x)^{\cancel{2}}}$$

$$= \frac{2 - 4x\cdot\ln 2 + (\ln 2)^2 x^2}{2^x} = 0$$

Quadratic Formula

$$x = \frac{4\ln 2 \pm \sqrt{(4\ln 2)^2 - (4)(2)(\ln 2)^2}}{2(\ln 2)^2}$$

$$= \frac{4\cancel{\ln 2} \pm \cancel{\ln 2}\sqrt{16-8}}{2(\ln 2)^{\cancel{2}}} = \frac{\overset{2}{\cancel{4}} \pm \cancel{2}\sqrt{2}}{\cancel{2}\ln 2}$$

$$\boxed{x = \frac{2\pm\sqrt{2}}{\ln 2}}$$