IB Math HL2   Integration by Substitution (Work with your group)   Name: Key

1. Integrate by substitution.

a. $\int \frac{2x}{\sqrt{x^2-5}}dx$

$u = x^2-5$

$du = 2x\,dx$

$\int \frac{du}{\sqrt{u}} = \boxed{2\sqrt{x^2-5}+C}$

b. $\int \frac{\sin x}{\cos^4 x}dx$

$u = \cos x$

$du = \sin x\,dx$

$\int \frac{du}{u^4} = \boxed{\frac{1}{3\cos^3 x}+C}$

c. $\int 2x\sqrt{3x-1}\,dx$

$u = 3x-1 \rightarrow x = \frac{1}{3}(u+1)$

$\frac{1}{3}du = dx$

$\frac{2}{3}\int (u+1)u^{\frac{1}{3}}du$

$= \frac{2}{3}\int (u^{\frac{4}{3}} + u^{\frac{1}{3}})du$

$= \boxed{\frac{4}{45}(3x-1)^{5/2} + \frac{4}{27}(3x-1)^{3/2} + C}$

d. $\int \frac{1}{x}\cos(\ln x)dx$

$u = \ln x$

$du = \frac{1}{x}dx.$

$\int \cos u\,du$

$= \boxed{\sin(\ln x)+C}$

e. $\int \frac{\sin x - \cos x}{\cos x + \sin x}dx$

$u = \cos x + \sin x$

$du = \sin x - \cos x\,dx$

$\boxed{\ln|\cos x + \sin x| + C}$

f. $\int \sec x\,dx$

$\int \frac{\sec x\,dx}{1.} \frac{(\sec x + \tan x)}{(\sec x + \tan x)}$

$= \int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x}\cdot dx.$

$u = \sec x + \tan x$

$du = (\sec x \cdot \tan x + \sec^2 x)dx.$

$= \int \frac{du}{u} = \boxed{\ln|\sec x + \tan x| + C}$

g. $\int \cos^2(x)\sin^2(x)dx$ (Use trig identities).

$= \int \frac{1}{2}(1+\cos 2x)\cdot\frac{1}{2}(1-\cos 2x)dx$

$= \int \frac{1}{4}(1-\cos^2 2x)dx$

$= \int \frac{1}{4}(1 - \frac{1}{2}(1+\cos 4x)dx$

$= \int \frac{1}{4}(1-\frac{1}{2}-\frac{1}{2}\cos 4x)dx$

$= \int (\frac{1}{8} - \frac{1}{8}\cos 4x)dx$

$= \boxed{\frac{1}{8}x - \frac{1}{32}\sin 4x + C}$

$\frac{1}{8}x - \frac{1}{32}[2\sin 2x \cdot \cos 2x] + C$

$= \boxed{\frac{1}{8}x - \frac{1}{16}2\sin x\cos x(1-2\sin^2 x) + C}$

$= \boxed{\frac{1}{8}x - \frac{1}{8}\sin x\cos x(1-2\sin^2 x) + C}$