IB Math HL1 18C The Product Rule

Warm Up: Find $\dfrac{dy}{dx}$

1. $y=\left(5x^2-7x+2\right)^9$

$$\frac{dy}{dx}=9(5x^2-7x+2)^8(10x-7)$$

2. $y=\dfrac{8}{\sqrt[3]{4x^5+2x}}=8(4x^5+2x)^{-\frac{1}{3}}$

$$=\frac{-8}{3}(4x^5+2x)^{-\frac{4}{3}}(20x^4+2)$$

$$=-\frac{8(20x^4+2)}{(4x^5+2x)^{4/3}}$$

OR

$$=-\frac{8(20x^4+2)}{(4x^5+2x)\sqrt[3]{4x^5+2x}}$$

The Product Rule Investigation

Complete the table, finding $f'(x)$ by direct differentiation.

$f(x)$	$f'(x)$	$u(x)$	$v(x)$	$u'(x)$	$v'(x)$	$u'(x)v(x)+u(x)v'(x)$
x^2	$2x$	x	x	1	1	$1\cdot x+x\cdot 1=2x$
$x^{\frac{3}{2}}$	$\frac{3}{2}x^{\frac{1}{2}}$	x	$\sqrt{x}$	1	$\frac{1}{2}x^{-\frac{1}{2}}$	$\frac{3}{2}x^{\frac{1}{2}}$
$x(x+1)$	$2x+1$	x	$x+1$	1	1	$2x+1$
$(x-1)(2-x^2)$	$2-3x^2+2x$	$x-1$	$2-x^2$	1	$-2x$	$2-3x^2+2x$

$\hookrightarrow 2x-x^3-2+x^2$

The Product Rule

If $f(x)=u(x)\cdot v(x)$, then $f'(x)=u'(x)\cdot v(x)+u(x)\cdot v'(x)$

$$\frac{df}{dx}=\frac{du}{dx}\cdot v+u\cdot\frac{dv}{dx}$$

3. Given $y=\left(3x^2-4x\right)x^5$, find $\frac{dy}{dx}$ by

a. expanding first.

$y=3x^7-4x^6$

$\frac{dy}{dx}=21x^6-24x^5$

b. using the Product Rule. $y=u\cdot v \Rightarrow \frac{dy}{dx}=u'\cdot v+u\cdot v'$

$y=(3x^2-4x)\cdot x^5$

$\frac{dy}{dx}=(3x^2-4x)'\cdot x^5+(3x^2-4x)\cdot x^{5\,\prime}$

$=(6x-4)x^5+(3x^2-4x)(5x^4)$

$=6x^6-\boxed{4x^5}+15x^6-\boxed{20x^5}=21x^6-24x^5$

4. Find $\frac{dy}{dx}$ if $y=\sqrt{x}\left(2x-8x^7\right)^3$

$\frac{dy}{dx}=(\sqrt{x})'(2x-8x^7)^3+\sqrt{x}\left((2x-8x^7)^3\right)'$

$=\left(\frac{1}{2}x^{-\frac{1}{2}}\right)(2x-8x^7)^3+\sqrt{x}(3)(2x-8x^7)^2(2-56x^6)$

$=\frac{(2x-8x^7)^3}{2\sqrt{x}}+3\sqrt{x}(2x-8x^7)^2(2-56x^6)$

Practice) Use of power rule, chain rule, and product rule.

Find $\frac{df}{dx}$.

1. $f(x) = \frac{2}{\sqrt[4]{2x-5}} = 2(2x-5)^{-\frac{1}{4}}$

$$\frac{df}{dx} = (2)\left(-\frac{1}{4}\right)(2x-5)^{-\frac{5}{4}}(2)$$

$$= \boxed{\frac{-1}{(2x-5)^{5/4}}} \text{ OR } \boxed{\frac{-1}{(2x-5)\sqrt[4]{2x-5}}}$$

2. $f(x) = (x^3 - 5x)\sqrt{2x-3}$

$$\frac{df}{dx} = (3x^2-5)\sqrt{2x-3} + \frac{(x^3-5x)\cancel{2}\cancel{\tfrac{1}{2}}}{\sqrt{2x-3}}$$

$$= \boxed{(3x^2-5)\sqrt{2x-3} + \frac{(x^3-5x)}{\sqrt{2x-3}}}$$

Find the gradient of the tangent line to:

3. $f(x) = x^3\sqrt{5-3x}$ at $x = -2$

$$\frac{df}{dx} = 3x^2\sqrt{5-3x} + \frac{x^3\cdot\left(\frac{1}{2}\right)(-3)}{\sqrt{5-3x}}$$

$$\left.\frac{df}{dx}\right|_{x=(-2)} = f'_{(-2)} = (3)(-2)^2\sqrt{5+6} + \frac{\overset{-4}{\cancel{(-2)^3}}\left(\frac{1}{\cancel{2}}\right)(-3)}{\sqrt{5+6}}$$

$$= 12\sqrt{11} + \frac{12}{\sqrt{11}} = \frac{132+12}{\sqrt{11}} = \boxed{\frac{144}{\sqrt{11}}} \text{ OR } \boxed{\frac{144\sqrt{11}}{11}}$$

4. Suppose $y = \frac{a}{\sqrt{1+bx}}$ where a and b are constants. Find a and b given that $y(3) = 1$ and $y'(3) = -\frac{1}{8}$.

$$y(3) = \boxed{\frac{a}{\sqrt{1+3b}} = 1} \text{ ①} \Rightarrow a = \sqrt{1+3b} \qquad a = \sqrt{1+3} \Rightarrow \boxed{a=2}$$

$$y = a(1+bx)^{-\frac{1}{2}}$$

$$y' = (a)\left(-\frac{1}{2}\right)(1+bx)^{-\frac{3}{2}}(b) = \frac{-ab}{2\sqrt{1+bx}\,(1+bx)}$$

$$y'(3) = \frac{-ab}{2\sqrt{1+3b}\,(1+3b)} = -\frac{1}{8} \Rightarrow \frac{+b}{1\cdot\cancel{2}(1+3b)} = +\frac{1}{\cancel{8}\,4}$$

$\frac{a}{\sqrt{1+3b}} = 1$ Equation 1.

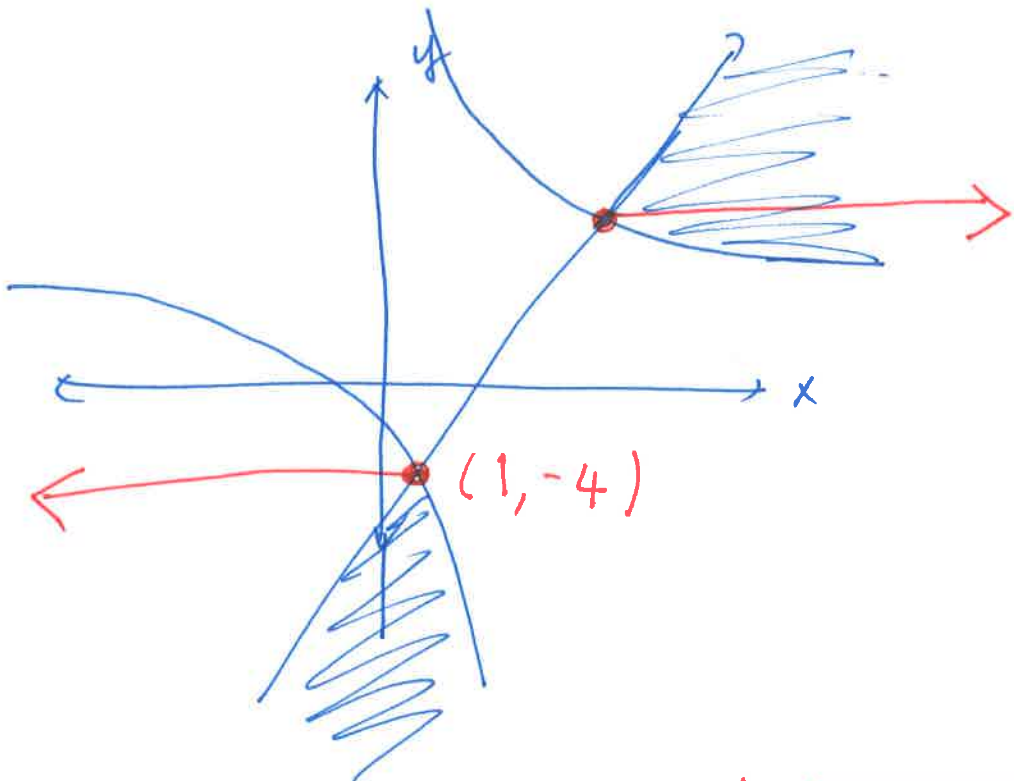
$$\Rightarrow 4b = 1 + 3b \qquad \boxed{b=1}$$

Solve $\dfrac{3x+1}{x-2} \le 2x-6$

$$\Rightarrow \quad \frac{3x+1}{x-2} \le y \qquad (1)$$

$$\Rightarrow \quad y \le 2x-6 \qquad (2)$$

Enter (1) & (2)



$(-\infty, 1) \cup (5.5, \infty)$