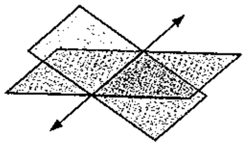
IB MM3 3-D Intersecting Planes **Name** ________________

- **Two planes** in space could have any of the following three arrangements:

(1) intersecting



(2) parallel

(3) coincident

(1) Solution: (line) $\dfrac{x-a}{x_1} = \dfrac{y-b}{y_1} = \dfrac{z-c}{z_1}$

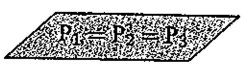
(2) No solution

(3) Solution: (plane) $Ax + By + Cz = D$

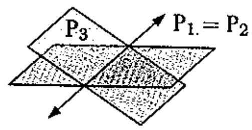
$A_1x + B_1y + C_1z = D_1$
$A_2x + B_2y + C_2z = D_2$ $\Rightarrow$ Find the directional vector by $\begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$

- **Three planes** in space could have any of the following eight arrangements:

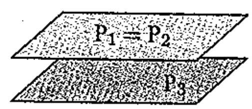
(1) all coincident



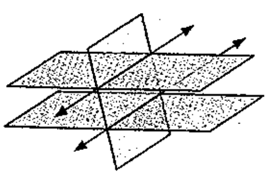
(2) two coincident and one intersecting



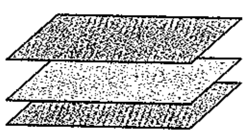
(3) two coincident and one parallel



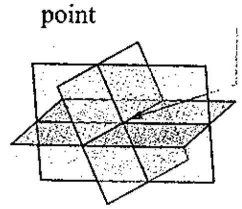
(4) two parallel and one intersecting



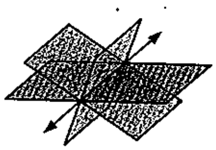
(5) all three parallel



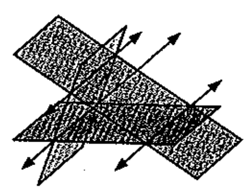
(6) all meet at the one point



(7) all meet in a common line



(8) the line of intersection of any two is parallel to the third plane.



- (6): unique solution

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \end{array}\right]$$

- (2) and (7): ∞ solutions

$$\left[\begin{array}{ccc|c} 1 & m & n & d_1 \\ 0 & 1 & k & d_2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

- The Rest: No solution

$$\left[\begin{array}{ccc|c} 1 & m & n & d_1 \\ 0 & 1 & k & d_2 \\ 0 & 0 & 0 & d_3 \end{array}\right]$$

USING ROW OPERATIONS TO SOLVE A 3×3 SYSTEM

A general 3×3 system in variables x, y, and z has the form $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$

where the coefficients of x, y, and z are constants.

$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$ is the system's **augmented matrix form** which we need to reduce to **echelon form**: $\Rightarrow$ $\left[\begin{array}{ccc|c} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \end{array}\right]$ using **row operations.**