6.2 Applications

1. Write and solve the differential equation that models this statement: the rate of change of *y* with respect to *t* is proportional to *y*. Solve for *y*.

$$\frac{dy}{dt} = ky$$

$$\int \frac{1}{y}\, dy = \int k\, dt$$

$$\ln y = kt + C_1$$

$$y = e^{kt + C_1}$$

$$y = e^{kt + C_1} = e^{kt} e^{C_1}$$

$$\boxed{y = Ce^{kt}}$$

C_1 and C are different

This is the most useful form of the equation.

The Law of Exponential Growth/Decay:

If $\frac{dy}{dt} = ky$ for some constant *k*, then $y = Ce^{kt}$.

C is the initial value of *y* and *k* is the **proportionality constant.**

Growth occurs when $k > 0$. Decay occurs when $k < 0$.

2. The rate of decay of a radioactive substance is proportional to the amount of substance present. Suppose 10 grams of the plutonium isotope ^{239}Pu, which has a half-life of 24,100 years, was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?

$y = Ce^{kt}$

$5 = 10e^{k(24100)}$

$k = -.00002876$

$y = 10e^{-.00002876t}$

$1 = 10e^{-.00002876t}$

$t = \boxed{80058 \text{ years}}$

3. Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Approximately how many flies were in the original population?

$y = Ce^{kt}$

$(2, 100)$ $\quad$ $(4, 300)$

$100 = Ce^{2k}$ $\quad$ $300 = Ce^{4k}$

$\frac{100}{e^{2k}} = C$ $\quad$ $300 = \frac{100}{e^{2k}} \cdot e^{4k}$

$3 = e^{2k}$

$\frac{100}{e^{2(.549306)}} = C$ $\quad$ $k = 0.549306$

$C \approx \boxed{33 \text{ fruit flies}}$

Newton's Law of Cooling states that the rate of cooling of a body is proportional to the difference in temperature between the body and its surroundings. If T is the temperature at time t minutes and T_s is the surrounding temperature, then $\frac{dT}{dt} = k(T - T_s)$.

4. A body is placed in a medium that is kept at a constant temperature of 20° C. The body cools from 70° C to 50° C in 10 minutes.

a. Find the function T(t) which measures the temperature after t minutes.

b. Find the temperature of the body after 20 minutes.

c. How long will it take the body to cool to half its initial temperature?

a. $\frac{dT}{dt} = k(T-20)$

$\int \frac{1}{T-20}\, dT = \int k\, dt$

$\ln(T-20) = kt + C_1$

$T - 20 = e^{kt+C_1}$

$T = 20 + Ce^{kt}$

$T(0) = 70^\circ$

$70 = 20 + Ce^{0}$

$50 = C$

$T(10) = 50^\circ$

$50 = 20 + 50e^{10k}$

$\frac{30}{50} = e^{10k}$

$k = -.05108$

$\boxed{T(t) = 20 + 50e^{-.05108t}}$

b. $T(20) = 20 + 50e^{-.05108(20)}$

$\approx \boxed{38^\circ C}$

c. $35 = 20 + 50e^{-.05108t}$

$\frac{15}{50} = e^{-.05108t}$

$t \approx \boxed{23.6 \text{ minutes}}$