

IB Math HL1: Mixed Review with all rules.

Name: key Period: \_\_\_\_\_

Find the derivatives of the following functions.

a)  $f(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{\frac{1}{2}} + 3x^{\frac{1}{3}}$

$$\frac{df}{dx} = 3x^{-\frac{1}{2}} + x^{-\frac{2}{3}} = \boxed{\frac{3}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}}$$

2.  $f(x) = 5x\sqrt{x^2-16} = 5x(x^2-16)^{\frac{1}{2}}$

$$\frac{df}{dx} = 5(x^2-16)^{\frac{1}{2}} + (5x)\left(\frac{1}{2}\right)(2x)(x^2-16)^{-\frac{1}{2}}$$

$$= \boxed{5\sqrt{x^2-16} + \frac{5x^2}{\sqrt{x^2-16}}} \text{ OR } \boxed{\frac{10x^2-80}{\sqrt{x^2-16}}}$$

b)  $f(t) = \frac{6t-5}{t^2+1}$

$$\frac{df}{dt} = \frac{6(t^2+1) - (6t-5)(2t)}{(t^2+1)^2}$$

$$= \frac{6t^2+6-12t^2+10t}{(t^2+1)^2} = \boxed{\frac{-6t^2+10t+6}{(t^2+1)^2}}$$

4.  $f(x) = \frac{3x}{\sqrt{x^2+1}}$

$$\frac{df}{dx} = \frac{3\sqrt{x^2+1} - \left(\frac{3}{\sqrt{x^2+1}}\right)(2x)(3x)(x^2+1)^{-\frac{1}{2}} \cdot \sqrt{x^2+1}}{x^2+1}$$

$$= \frac{3(x^2+1) - 3x^2}{(x^2+1)\sqrt{x^2+1}} = \boxed{\frac{3}{(x^2+1)^{3/2}}}$$

5.  $f(x) = \left(\frac{x+5}{x^2+3}\right)^2$

$$\frac{df}{dx} = 2\left(\frac{x+5}{x^2+3}\right) \left(\frac{(x^2+3) - (x+5)(2x)}{(x^2+3)^2}\right) = \frac{(2x+10)(x^2+3-2x^2-10x)}{(x^2+3)^3}$$

$$= \boxed{\frac{(2x+10)(3-x^2-10x)}{(x^2+3)^3}}$$

6. Find the gradient of the tangent to the curve of

a)  $f(x) = x\sqrt{x^2+1}$  at  $(3, 3\sqrt{10})$

$$\frac{df}{dx} = \sqrt{x^2+1} + x\left(\frac{1}{2}\right)(2x)(x^2+1)^{-\frac{1}{2}}$$

$$\left.\frac{df}{dx}\right|_{x=3} = \sqrt{9+1} + \frac{(3)^2}{\sqrt{9+1}} = \sqrt{10} + \frac{9}{\sqrt{10}} = \boxed{\frac{19}{\sqrt{10}}} \text{ OR } \boxed{\frac{19\sqrt{10}}{10}}$$

b)  $f(x) = \frac{4x^2}{(1-x)^2}$  at  $(2, 16)$

$$\frac{df}{dx} = \frac{8x(1-x)^2 + (4x^2)(2)(1-x)}{(1-x)^4}$$

$$\left.\frac{df}{dx}\right|_{x=2} = \frac{(8)(2)(1-2)^2 + (4)(2)^2(2)(2-1)}{(1-2)^4} = 16 + 32 = \boxed{48}$$