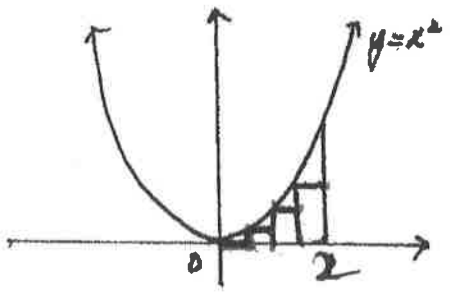
① 0

Warm UP

Find the area between the parabola $y = x^2$ and the x-axis on the interval [0, 1] with 4 rectangles;

(1) Using Left-hand rectangles. width : $\Delta x = \frac{1}{2}$

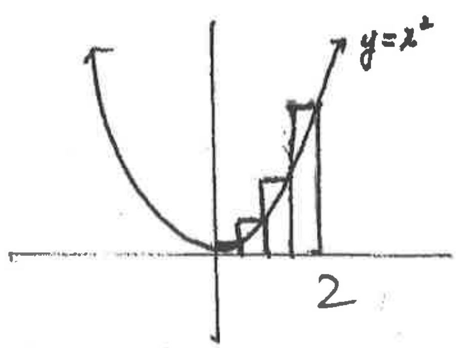



$\text{Area} = \frac{1}{2}[f(0) + f(\frac{1}{2}) + f(1) + f(\frac{3}{2})]$

$= \frac{1}{2}[0^2 + (\frac{1}{2})^2 + (1)^2 + (\frac{3}{2})^2]$

$\approx \frac{7}{4} \approx 1.75$

(2) Using Right-Hand Rectangles.




$\text{Area} = \frac{1}{2}[f(\frac{1}{2}) + f(1) + (\frac{3}{2}) + f(2)]$

$= \frac{1}{2}[\cancel{x^2} + (\frac{1}{2})^2 + (1)^2 + (\frac{3}{2})^2 + 2^2]$

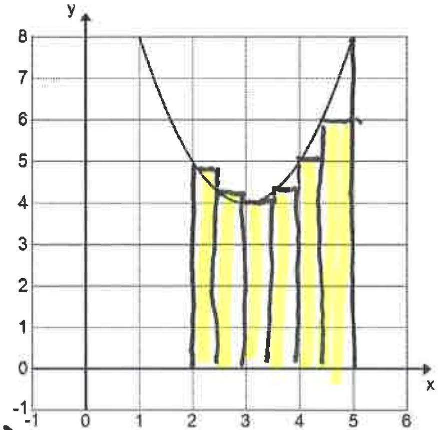
$\approx 3.75$

**The first Fundamental Theorem of Calculus:**

1. **Given** $f(x) = x^2 - 6x + 13$ on the interval [2, 5] using the indicated number of rectangles.

a. Sketch the 6 left hand rectangles under the curve on the interval [2, 5] .

$$\Delta x = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$$




$f(x) = x^2 - 6x + 13$

Left-Hand

$$\sum_{k=0}^{n-1} \Delta x\, f(a + \Delta x k)$$

Right-Hand

$$\sum_{k=1}^{n} \Delta x\, f(a + \Delta x k)$$

b. Estimate the area using the indicated number of rectangles.

| Number of rectangles | Left-hand rectangles. Show Sigma Notation | Area | Right-hand rectangles Show Sigma Notation | Area |
|---|---|---|---|---|
| 6 $\Delta x = \frac{1}{2}$ | $\sum_{k=0}^{5} \frac{1}{2} f(2 + \frac{1}{2}k)$ | 14.375 | $\sum_{k=1}^{6} \frac{1}{2} f(2 + \frac{1}{2}k)$ | 15.875 |
| 72 $\Delta x = \frac{3}{72}$ | $\sum_{k=0}^{71} \frac{3}{72} f(2 + \frac{3}{72}k)$ | 14.9384 | $\sum_{k=1}^{72} \frac{3}{72} f(2 + \frac{3}{72}k)$ | 15.0634 |
| $\infty$ | $\lim_{n\to\infty} \sum_{k=0}^{n-1} \frac{3}{n} f(2 + \frac{3}{n}k)$ | 15 | $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n} f(2 + \frac{3}{n}k)$ | 15 |

c.

a. Find F(x), the antiderivative $f(x) = x^2 - 6x + 13$.

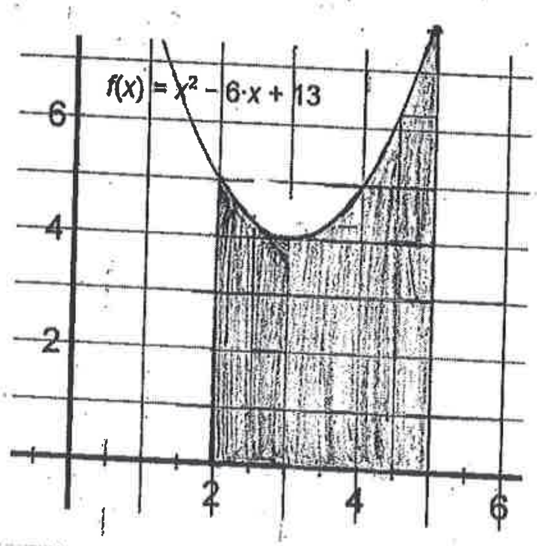
$$F(x) = \int (x^2 - 6x + 13)\,dx = \frac{1}{3}x^3 - \frac{6}{2}x^2 + 13x + C$$

b. Evaluate F(5)-F(2)

$$F(5) = \frac{5^3}{3} - (3)(5)^2 + 13(5) + C$$

$$- F(2) = \frac{2^3}{3} - (3)(2)^2 + 13(2) + C$$

$$\Rightarrow \boxed{15}$$




Area Under the curve using left-hand-rectangles n=∞

$$A(f, 2 \le x \le 5) = \frac{3}{\infty} f(2) + \frac{3}{\infty} f(2.0000000000001) \ldots\ldots\ldots\ldots\ldots\ldots$$

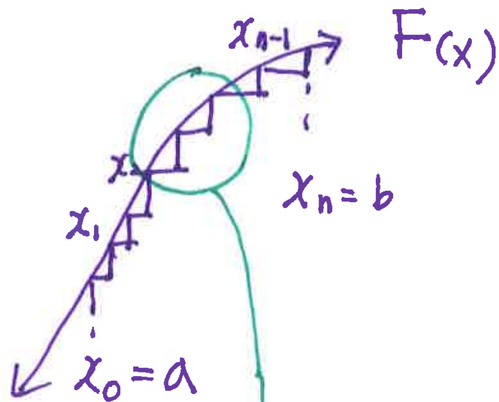
Nspire: $A(f, 2 \le x \le 5) = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{3}{n} f\left(2 + \frac{3}{n} k\right)$

TI 84: sum(seq((1/∞$Y_1$(2+(3/∞)K), K, 0, ∞, 1))

Use $n$ = 50,000 = 15
a Large #

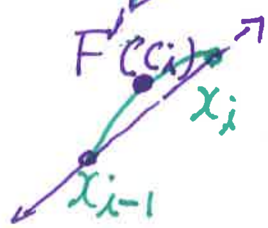
④

I; $a = x_0 < x_1 < x_2 \cdots < x_{n-1} < x_n = b$




$$
\begin{aligned}
F(b) - F(a) = {} & F(x_n) - F(x_{n-1}) \\
& + F(x_{n-1}) - F(x_{n-2}) \\
& \vdots \\
& + F(x_1) - F(x_0)
\end{aligned}
$$

$$\Rightarrow F(b) - F(a) = \sum_{i=1}^{n} F(x_i) - F(x_{i-1})$$




II. MVT:

$$F'(c_i) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$$

Let $(x_i - x_{i-1}) = \Delta x_i$

$$F'(c_i) = f(c_i)$$

$$\sum_{i=1}^{n} f(c_i) \cdot \Delta x_i = \sum_{i=1}^{n} F(x_i) - F(x_{i-1})$$

$= F(b) - F(a)$ from I.

$$\therefore \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \cdot \Delta x_i = F(b) - F(a)$$

Name______________________

**Antiderivative**

If $F'(x)=f(x)$, then $F(x)$ is an antiderivative of $f(x)$

**Indefinite Integral**

$\int f(x)\,dx = F(x)+C$

Represents all antiderivatives of $f(x)$

**Riemann Sum**

$$\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

Sum of n rectangles approximating the area under $f(x)$.

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

Sum of infinitely many rectangles of infinitesimal width is the **EXACT area** under $f(x)$ on the interval $[a,b]$.

**Fundamental Theorem of Calculus**

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x =$$

If $f(x)$ is continuous with antiderivative $F(x)$, then $\int_a^b f(x)\,dx = F(b)-F(a)$.

If a function $f$ is continuous on the closed interval $[a, b]$ and $F$ is an antiderivative of $f$ on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Example 1)

a. Use 20 right-hand rectangles to approximate $\int_2^7 (4x-12)\,dx$.

Sigma Notation: $\Delta x = \frac{7-2}{20} = \frac{1}{4}$

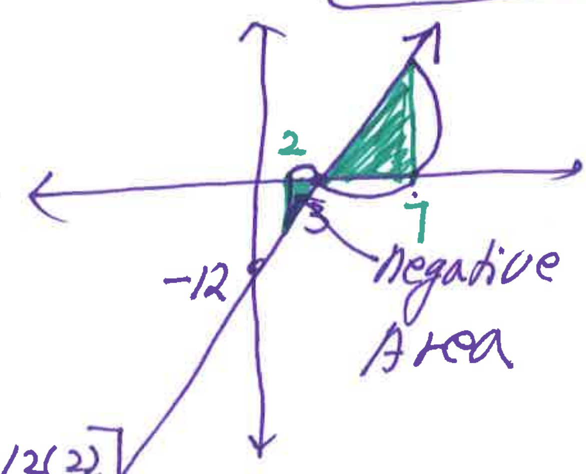
Rounded to 4 decimal places:

$$\sum_{k=1}^{20} \frac{1}{4} f\left(2+\frac{1}{4}k\right) \text{ where } f(x)=4x-12 \quad \boxed{32.5000}$$

b. Use graphical evidence to evaluate $\int_2^7 (4x-12)\,dx$.

$$-\frac{1}{2}(1\cdot 4)+\frac{1}{2}(4)(16)$$

$$-2+32 = \boxed{30}$$




b. Use the FTC to evaluate $\int_2^7 (4x-12)\,dx$.

$$\left[\frac{4}{2}x^2-12x\right]_{x=2}^{x=7}$$

$$=\left[2(7)^2-12(7)\right]-\left[2(2)^2-12(2)\right]$$

$$=\boxed{30}$$