Continuity

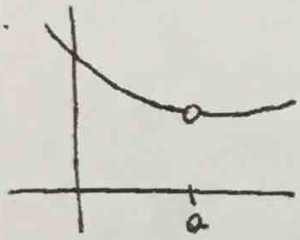
A function f is **continuous** at $x = a$ if and only if

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Example 1

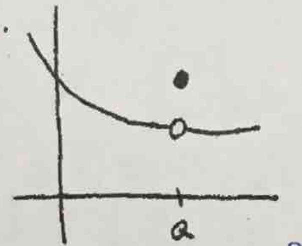
State whether the function is continuous or discontinuous at $x = a$. If it is discontinuous, state which condition of continuity fails.

A.



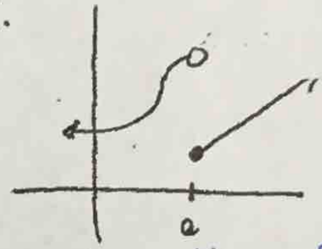
not continuous
condition #1 → $f(a)$ DNE

B.



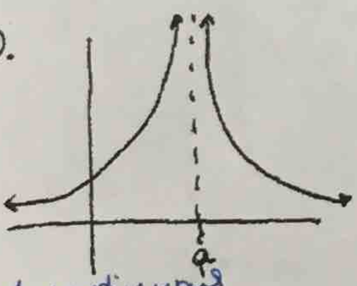
not continuous
#3 $\lim_{x \to a} f(x) \neq f(a)$

C.



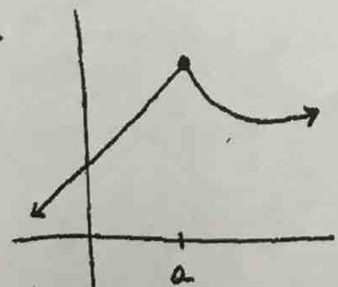
not continuous
#2 $\lim_{x \to a} f(x)$ DNE

D.



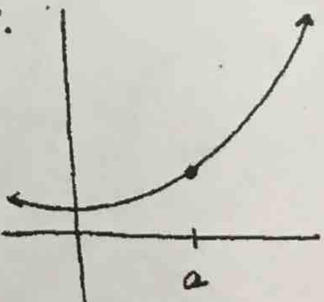
not continuous
#1 $f(a)$ DNE

E.



continuous

F.



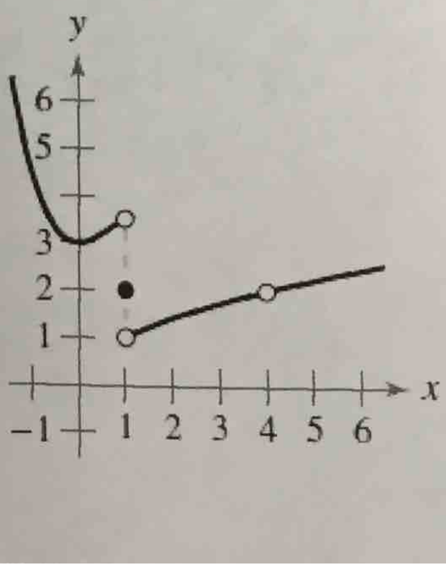
continuous

Example 2

Describe the interval(s) on which the function is continuous.

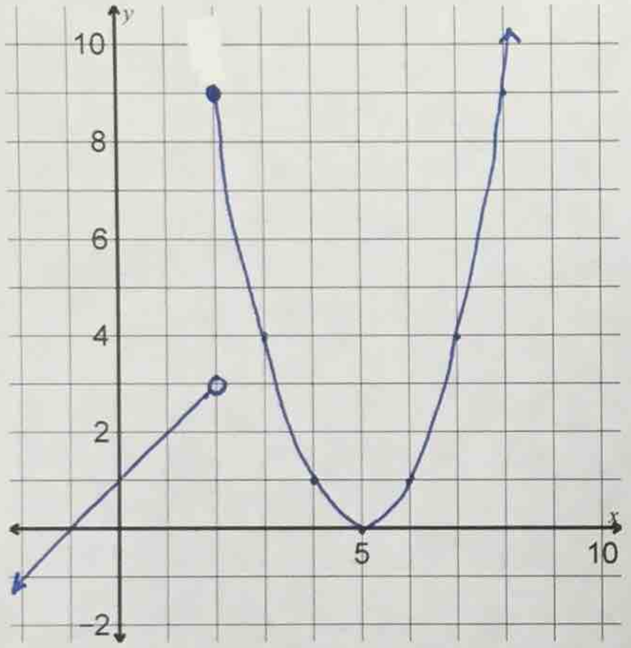
$\mathbb{R}, x \neq 1, 4$

$(-\infty, 1) \cup (1, 4) \cup (4, \infty)$



Example 3 $\qquad f(x)=\begin{cases} x+1, & x<2 \\ a(x-5)^2, & x \ge 2 \end{cases}$

a. Graph $f(x)$ if $a=1$.



b. State which condition(s) of continuity fail at $x=2$ in part a.

$\lim_{x \to 2} f(x)$ DNE

$f(x) = \begin{cases} x+1, & x<2 \\ (x-5)^2, & x \ge 2 \end{cases}$

c. Find the value of a for which $f(x)$ is continuous at $x=2$.

Step 1: Find each one-sided limit in terms of a.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2^-} (x+1) = \lim_{x \to 2^+} \left(a(x-5)^2\right)$

Step 2: Assume the limit exists and solve for a.

$2+1 = a(2-5)^2$

$3 = 9a$

$\boxed{\frac{1}{3} = a}$