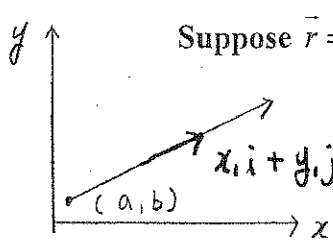


**The Velocity Vector of a Moving Object:**

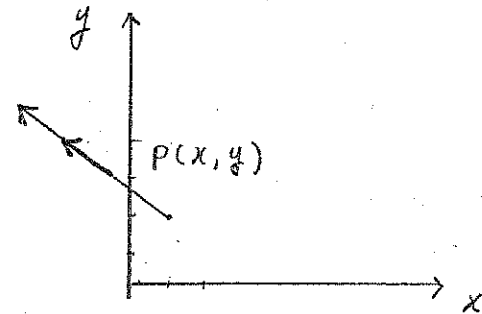
Suppose  $\vec{r} = \begin{pmatrix} a \\ b \end{pmatrix} + t \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  is a vector equation of the path of an object.



Initial Position:  $(a, b)$   
 Speed:  $\sqrt{x_1^2 + y_1^2}$   
 Velocity Vector:  $v = x_1i + y_1j$

**Example 1)** A particle at  $P(x(t), y(t))$  moves such that  $x(t) = 2 - 3t$  and  $y(t) = 4 + 2t$  where  $t \geq 0$ . Time is in seconds and distance is in meters.

a. Find the initial position of P?



b. Find the positions of P for  $t=1$  and  $t=2$ .

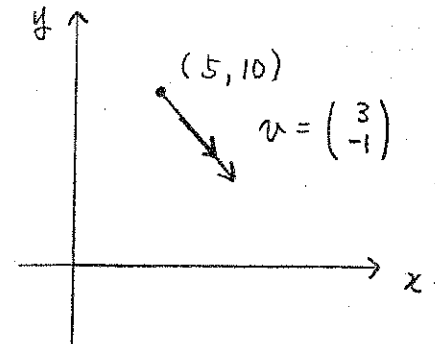
c. Find the speed (meters/second).

**Example 2)** An object is initially at  $(5, 10)$  and moves with velocity vector  $3i - j$  meters per minute.

a) Find the position of the object at time  $t$  minutes.

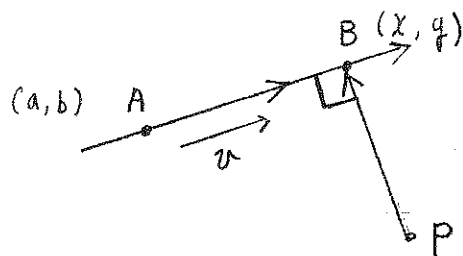
b) Find the position of the object at  $t=3$  minutes.

c) Find the time and the position when the object is due east of  $(0, 0)$ .



IB Math 2: (The Shortest Distance from a line to a point) –Day two

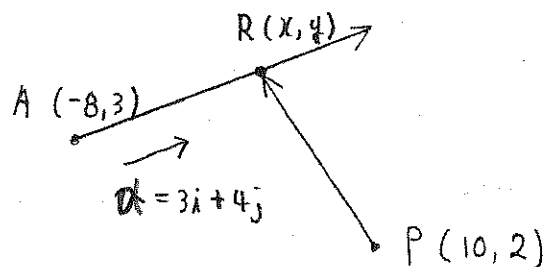
An object which is moving along with the line  $AB$  is closest to the given point  $P$  when  $\overline{AB}$  is perpendicular  $\overline{BP}$ .  $\Leftrightarrow \overline{AB} \cdot \overline{PB} = 0$



Warm Up (Review of HW) # 8 d) p 446

Example 3) If distances are measured in km and a ship R moves in the direction  $3i + 4j$  at speed of 10 km/hr from the initial position  $A (-8, 3)$ .

a) Find an expression for the ship in terms of  $t$  when  $t$  is time after leaving port A.



b) Find the time when the ship is closest to point  $P (10, 2)$ .

**Example 4)** Consider the point P (-1, 2, 3) and the line with  $\vec{r} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ .

a) Find the coordinates of the foot of the perpendicular from P to the line.

b) Find the shortest distance from the point to the line.

Practice) A line has vector equation  $\vec{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ , where  $t \geq 0$ . Let P be the point (5, -1). Find exactly the shortest distance from P to the line.

IB Math 2: Intersecting Lines and skewed lines - Day 3

Example 5) Given  $L_1: r_1 = -2\mathbf{i} + \mathbf{j} + t(3\mathbf{i} + 2\mathbf{j})$   
 $L_2: r_2 = 15\mathbf{i} + 5\mathbf{j} + \lambda(-4\mathbf{i} + 1\mathbf{j})$ ;

Using vector methods, find the point where the two lines meet.

$$L_1: x = -1 + 2\lambda, \quad y = 1 - 2\lambda, \quad z = 1 + 4\lambda$$

Example 6) Given  $L_2: r_2 = (\mathbf{i} + 3\mathbf{k}) + t(-\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

$$L_3: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

a) Show that line 1 and line 3 intersect and find the intersection.

b) Show that line 1 and line 3 are skewed.