key

Properties of Definite Integrals

1. a. $\int_1^4 3x^2 dx = 3\int_1^4 x^2 dx = 3\left[\frac{x^3}{3}\right]_{x=1}^{x=4}$

$= 4^3 - 1^3 = \boxed{63}$

b. $\int_4^1 3x^2 dx = -\int_1^4 3x^2 dx = \boxed{-63}$

$\int_b^a f(x)\,dx = -\int_a^b f(x)dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)dx$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)dx$

f(x) is continuous on $[a, c]$ and $a < b < c$.

$\int_a^b [f(x)+g(x)]\,dx =$

$\int_a^b f(x)dx + \int_a^b g(x)dx$.

2. $\int_1^4 15x^2 dx = 5\int_1^4 3x^2 dx = (5)(63)$

$= \boxed{315}$

3. $\int_1^2 3x^2 dx + \int_2^4 3x^2 dx = \int_1^4 3x^2 dx = \boxed{63}$

4. a. $\int_1^4 3x^2 dx + \int_1^4 2x dx = \int_1^4 (3x^2 + 2x)dx$

$= \left[x^3 + x^2\right]_{x=1}^{x=4} = \left[64+16\right] - \left[1+1\right]$

$= \boxed{78}$

b. $\int_1^4 (3x^2 + 2x)\,dx = \boxed{78}$

5. Given $\int_{-2}^{5} f(x)\,dx = 12$, $\int_{-2}^{1} f(x)\,dx = -2$, $\int_{-2}^{5} g(x)\,dx = 7$, find each of the following.

a. $\int_1^5 f(x)\,dx = \int_{-2}^{5} f(x)dx - \int_{-2}^{1} f(x)dx$

$= 12 - (-2) = \boxed{14}$

b. $\int_5^1 f(x)\,dx = -\int_1^5 f(x)dx = \boxed{-14}$

c. $\int_{-2}^{5} 3f(x)\,dx = 3\int_{-2}^{5} f(x)dx$

$= 3 \cdot 12 = \boxed{36}$

d. $\int_{-2}^{5} (3f(x) - 2g(x))\,dx$

$= 3\int_{-2}^{5} f(x)dx - 2\int_{-2}^{5} g(x)dx = (3)(12) - (2)(7)$

$= 36 - 14 = \boxed{22}$