Key.

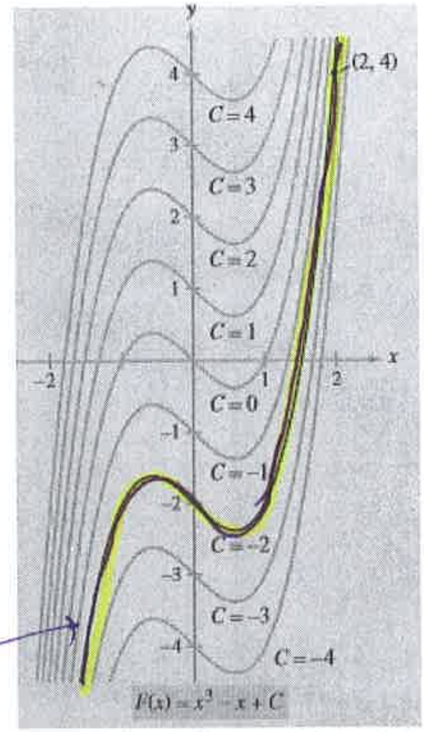
General Solutions vs. Particular Solutions

1. A General Solution Example)

Given $\frac{dy}{dx} = 3x^2 - 1$, find y.

$$y = \int (3x^2 - 1)\,dx = \frac{3}{3}x^3 - (1)(x) + C$$

$$= x^3 - x + C$$



The particular solution that satisfies the initial condition $F(2) = 4$ is $F(x) = x^3 - x - 2$.

2. A Particular Solution Example)

Given $\frac{dy}{dx} = 3x^2 - 1$ and $y(2) = 4$, find y.

$$y = x^3 - x + C$$

$$4 = (2)^3 - 2 + C$$

$$C = 4 - 8 + 2 = -2$$

$$\Rightarrow \boxed{y = x^3 - x - 2}$$

3. Find $f(x)$ where $\frac{df}{dx} = \cancel{\tan x}$ $\cos x$ and $f(\frac{\pi}{4}) = 0$

$$f(x) = \int (\cos x)\,dx = \sin x + C$$

$$0 = \sin(\tfrac{\pi}{4}) + C \Rightarrow C = -\sin(\tfrac{\pi}{4}) = -\frac{\sqrt{2}}{2}$$

$$\Rightarrow \boxed{f(x) = \sin x - \frac{\sqrt{2}}{2}}$$

4. Given the data $K''(x) = \cos(x) - 5e^x$, $K'(0) = 1$, and $K(0) = -2$, find $K(x)$.

$$K' = \int (K'')\,dx = \int (\cos x - 5e^x)\,dx = \sin x - 5e^x + C_1$$

$$\Rightarrow 1 = \sin 0 - 5e^0 + C_1 \Rightarrow C_1 = 6$$

$$K(x) = \int K'(x)\,dx = \int (\sin x - 5e^x + 6)\,dx = -\cos x - 5e^x + 6x + C_2$$

$$\Rightarrow -2 = -\cos(0) - 5 \cdot e^0 + C_2 \Rightarrow C_2 = 4$$

$$\boxed{K(x) = -\cos x - 5e^x + 6x + 4}$$