IB Math HL1: More Practice WS
(Anti-Derivatives and area under the curve)

Name: ANSWER KEY
Period: 4

1.

Differentiation	Integration
$\frac{d}{dx}(\frac{1}{3}x+\frac{1}{2x})=\boxed{\frac{1}{3}-\frac{1}{2x^2}}=\frac{2x^2-3}{6x^2}$ $\frac{d}{dx}(\frac{1}{3}x+\frac{1}{2}x^{-1})=\frac{1}{3}+\frac{1}{2}(-x^{-2})$	$\int\frac{2x^2-3}{6x^2}dx=\frac{1}{3}x+\frac{1}{2x}+C$
$\frac{d}{dx}(3\ln x-\frac{1}{2}e^{-4x})=\boxed{\frac{3}{x}+2e^{-4x}}$ $=\frac{3}{x}-\frac{1}{2}e^{-4x}\cdot -4=\frac{3}{x}+2e^{-4x}$	$\int(\frac{3}{x}+2e^{-4x})dx=3\ln x-\frac{1}{2}e^{-4x}+C$
$\frac{d}{dx}(-\frac{5}{3}\cos 3x)=\boxed{5\sin 3x}$ $=-\frac{5}{3}(-\sin 3x)\cdot 3=5\sin 3x$	$\int 5\sin 3x\,dx=-\frac{5}{3}\cos 3x+C$
$\frac{d}{dx}(\frac{1}{2}\tan 2x)=\boxed{\sec^2 2x}$ $=\frac{1}{2}(\sec^2 2x)\cdot 2=\sec^2 2x$	$\int\sec^2 2x\,dx=\frac{1}{2}\tan 2x+C$

2. Given $f(x)=2x^2-5$,

a) Find the area A of the region between y=f(x) and x-axis on the interval [1, 3] using left hand rectangle and n=20. Show proper sigma notation and use Graphing calculator to evaluate.

$$\frac{b-a}{n}=\frac{3-1}{20}=\frac{2}{20}=\frac{1}{10}$$

$$\sum_{k=0}^{19}\frac{1}{10}f(1+0.1k)=\boxed{6.54}$$

b) Find the area A of the region between y=f(x) and x-axis on the interval [1, 3] using right hand rectangle and n=20. Show proper sigma notation and use Graphing calculator to evaluate.

$$\sum_{k=1}^{20}\frac{1}{10}f(1+0.1k)=\boxed{8.14}$$

c) Find the exact area of A using the Fundamental Theorem of calculus.

$f(x)=2x^2-5$

$F(x)=\int f(x)dx=\frac{2}{3}x^3-5x+C$

$F(3)-F(1)=\left(\frac{2}{3}(3)^3-5(3)\right)-\left(\frac{2}{3}(1)^3-5(1)\right)=(18-15)-(\frac{2}{3}-5)=3-(-\frac{13}{3})=\boxed{\frac{22}{3}}$

Find the indefinite integral (Anti-derivatives).

2. $\int(\sqrt[3]{x}+2e^{-4x})dx$

$\int(x^{\frac{1}{3}}+2e^{-4x})\,dx$

$\boxed{\frac{3}{4}x^{\frac{4}{3}}-\frac{1}{2}e^{-4x}+C}$

3. $\int\frac{5}{3x-1}dx$

$\int 5(3x-1)^{-1}\,dx$

$\boxed{\frac{5}{3}\ln(3x-1)+C}$

4. $\int\frac{x^2-3}{\sqrt{x}}dx$

$\int\left(\frac{x^2}{\sqrt{x}}-\frac{3}{\sqrt{x}}\right)dx$

$\int(x^{\frac{3}{2}}-3x^{-\frac{1}{2}})\,dx$

$\boxed{\frac{2}{5}x^{\frac{5}{2}}+6x^{\frac{1}{2}}+C}$

5. $\int\frac{5}{(x+1)^4}dx$

$\int 5(x+1)^{-4}\,dx$

$\boxed{-\frac{5}{3}(x+1)^{-3}+C}$

6. $\int\sin(5x+2)dx$

$\boxed{-\frac{1}{5}\cos(5x+2)+C}$

7. Given $\frac{dy}{dx}=\sec^2 x$ and y(0)=3. Find the y(x).

$\int\sec^2 x = \tan x + C$

$y(0)=3=\tan(0)+C$

$3=0+C$

$3=C$

$\boxed{y(x)=\tan x+3}$