Differentiation and Integration

Exit slip Answers

#1. $\int e^{-3x}\cos 2x\,dx = A$

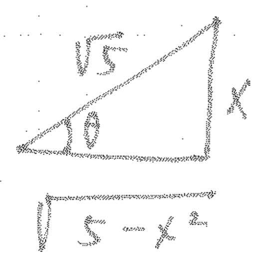
u	dx
e^{-3x}	$\cos 2x$
e^{-3x} (+)	$\frac{1}{2}\sin 2x$
$9e^{-3x}$ (−)	$-\frac{1}{4}\cos 2x$
(+)	

$$= \frac{1}{2}e^{-3x}\sin 2x - \frac{3}{4}e^{-3x}\cos 2x - \frac{9}{4}\underbrace{\int e^{-3x}\cos 2x\,dx}_{=A}$$

$$\Rightarrow \frac{13}{4}A = \left[\frac{1}{2}e^{-3x}\sin 2x - \frac{3}{4}\cos 2x\, e^{-3x}\right]$$

$$A = \boxed{\frac{2}{13}e^{-3x}\sin 2x - \frac{3}{13}e^{-3x}\cos 2x + C}$$

#2. $\int \sqrt{5-x^2}\,dx$



$\sin\theta = \frac{x}{\sqrt{5}}$

$dx = \sqrt{5}\cos\theta\,d\theta$

$\sqrt{5}\cos\theta = \sqrt{5-x^2}$

$$= \int 5\cos^2\theta\,d\theta$$

$$= \frac{5}{2}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{5}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C = \frac{5}{2}\left[\sin^{-1}\frac{x}{\sqrt{5}} + \left(\frac{x}{\sqrt{5}}\right)\left(\frac{\sqrt{5-x^2}}{\sqrt{5}}\right)\right] + C$$

$$= \boxed{\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{1}{2}x\sqrt{5-x^2} + C}$$

2

#3. $\int \tan^{-1}(5x)\,dx$

$= x\tan^{-1}5x - \int \frac{5x}{1+25x^2}\,dx$

$\boxed{\begin{array}{l|l} u=\tan^{-1}(5x) & dv = 1dx \\ du = \frac{5}{1+25x^2} & v = x \end{array}}$

$\begin{pmatrix} u = 1+25x^2 \\ du = 50x\,dx \\ \frac{du}{10} = 5x\,dx \end{pmatrix}$

$= x\tan^{-1}5x - \frac{1}{10}\ln(1+25x^2) + C$

#4. $\int \frac{\sqrt{x^2-9}}{x}\,dx$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-9} = 3\tan\theta$

$\int \frac{3\tan\theta}{\cancel{3}\sec\theta}\cdot\cancel{3}\sec\theta\tan\theta\,d\theta$

$= \int 3\tan^2\theta\,d\theta = 3\int(\sec^2\theta - 1)\,d\theta = 3[\tan\theta - \theta] + C$

$= 3\left(\frac{\sqrt{x^2-9}}{3} - \sec^{-1}\left(\frac{x}{3}\right)\right] + C$

#5 $\int_{-1}^{2} \frac{5x}{\sqrt{x+2}}\,dx$

$\begin{pmatrix} u = x+2 \\ du = dx \end{pmatrix}$

$\begin{pmatrix} x = u-2 \\ x=-1 \quad u=1 \\ x=2 \quad u=4 \end{pmatrix}$

$= 5\int_1^4 (u^{\frac{1}{2}} - 2u^{-\frac{1}{2}})\,du$

$\frac{2}{3}\cdot 8 \quad \frac{16}{3} - \frac{2}{3}$

$= 5\left[\frac{2}{3}u^{\frac{3}{2}} - 4\sqrt{u}\right]_1^4 = 5\left(\frac{2}{3}\cdot 4^{\frac{3}{2}} - \cancel{4\sqrt{4}} - \frac{2}{3} \cancel{+4}\right) = \cancel{10}$

$\boxed{\frac{10}{3}}$

③

#6

① $x=1 \Rightarrow 4y^2=9 \Rightarrow y=\pm\frac{3}{2} \Rightarrow y>0 \Rightarrow \boxed{y=\frac{3}{2}}$

Find

② $8y\ln x - 2x^2 + 4y^2 = 7$

$8\frac{dy}{dx}\cdot\ln x + 8y\cdot\frac{1}{x} - 4x + 8y\cdot\frac{dy}{dx} = 0$

substitute in $x=1 \quad y=\frac{3}{2}$

$\Rightarrow 8\cdot\frac{dy}{dx}\ln(1) + (4)(\frac{3}{2})(1) - 4 + 8(\frac{3}{2})\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{3}$

③ $(1, \frac{3}{2}) \quad m = \frac{2}{3} \Rightarrow \boxed{y - \frac{3}{2} = -\frac{2}{3}(x-1)}$

#7. $V = \pi\int_0^{12}\left(\frac{x^{1/2}}{\sqrt{x^2+9}}\right)^2 dx = \pi\int_0^{12}\frac{x}{x^2+9}dx$

$\begin{pmatrix} u = x^2+9 & x=0 \Rightarrow u=9 \\ du = 2x\,dx & x=12 \Rightarrow u=153 \end{pmatrix}$

$\frac{\pi}{2}\int_9^{153} u^{-1}du = \frac{\pi}{2}\ln u\Big|_{u=9}^{u=153}$

$= \frac{\pi}{2}(\ln 153 - \ln 9) = \boxed{\frac{\pi}{2}\ln(17)}$

#8

a) $f(x) = 1 - 2x^{-\frac{1}{3}} = 0 \Rightarrow \sqrt[3]{x} = 2$ $\boxed{x = 8}$

2nd derivative test:

b) $f''(x) = \frac{2}{3} x^{-4/3} = \frac{\quad}{3x\sqrt[3]{x}}$ $f''(8) > 0$

$x = 8$ is minimum point

$y = -4$ $\boxed{(8, -4)}$

OR 1st derivative test

$x = 8$

$f'(x)$ $-$ $+$

$8 = x$ is min.