WS. #1 Solutions.

#1. $y = \pm\sqrt[4]{7x^4 + D}$

#2. $y = \pm\sqrt{C - \frac{5}{2}x^2}$

#3. $y = \pm\sqrt{2\tan^{-1}x + D}$

#4. See attached.

#5. $\int \frac{x\,dx}{x^2+1} = \int \frac{1}{3-2y}\,dy$

$\frac{1}{2}\ln(x^2+1) + C = -\frac{1}{2}\ln|3-2y|$

$\ln\left(\frac{1}{x^2+1}\right) + C = \ln|3-2y|$ $\quad C = \ln A$

$\Rightarrow \ln\left|\frac{A}{x^2+1}\right| = \ln|3-2y|$

$\Rightarrow 3-2y = \frac{A}{x^2+1}$

$-2y = \frac{A}{x^2+1} - 3 \Rightarrow \boxed{y = \frac{3}{2} - \frac{A}{2(x^2+1)}}$

#6. $y^2\,dy = e^x\,dx$

$\frac{1}{3}y^3 = e^x + C$ $\quad (3C = D)$.

$y = \sqrt[3]{3e^x + D}$ ✓

#4. $x^2(y^2+1)\,dx + y\sqrt{x^3+1}\,dy = 0$

$x^2(y^2+1)\,dx = -y\sqrt{x^3+1}\,dy$

$$\int \frac{x^2\,dx}{\sqrt{x^3+1}} = \int \frac{-y\,dy}{y^2+1}$$

$u = x^3+1$

$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^{-\frac{1}{2}}\,du$

$u = y^2+1$

$\frac{1}{2}du = y\cdot dy$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$\Rightarrow \frac{2}{3}(x^3+1)^{\frac{1}{2}} + C = -\frac{1}{2}\ln(y^2+1)$

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$(-2)\left[\left(\frac{2}{3}\right)(x^3+1)^{\frac{1}{2}} + C\right] = (-2)\left(-\frac{1}{2}\right)\ln(y^2+1)$

$\Rightarrow -\frac{4}{3}(x^3+1)^{\frac{1}{2}} - 2C = \ln(y^2+1)$

$\Rightarrow y^2+1 = e^{-\frac{4}{3}\sqrt{x^3+1} - 2C}$ $\quad (e^{-2C} = A)$

$\Rightarrow y^2+1 = A\cdot e^{-\frac{4}{3}\sqrt{x^3+1}}$

$\Rightarrow y = \pm\sqrt{A\cdot e^{-4/3\sqrt{x^3+1}} - 1}$

#7. $x \sin x \, dx = -\cos 2y \, dy$.

$\Rightarrow -x\cos x + \sin x + C = \frac{-1}{2}\sin 2y$ $\quad (D = -2C)$

$\Rightarrow \sin 2y = 2x\cos x - 2\sin x + D$

$\Rightarrow 2y = \arcsin\left[2x\cos x - 2\sin x + D\right]$

$y = \frac{1}{2}\arcsin\left[2x\cos x - 2\sin x + D\right]$

#8. $\frac{\ln x}{x} dx = \frac{1}{y} dy$

$u = \ln x$, $du = \frac{1}{x}dx$ $\rightarrow$ $\frac{1}{2}(\ln x)^2 + C = \ln|y|$

$y = e^{\frac{1}{2}(\ln x)^2 + C} = \boxed{A \cdot e^{\frac{1}{2}(\ln x)^2}}$ $\quad (e^C = A)$.

#9. $y^{-2}dy = x^{-3}dx$

$\frac{-1}{y} = \frac{-1}{2x^2} + C$. $\quad \in y(1) = 2$.

$\Rightarrow \frac{-1}{2} = \frac{-1}{2} + C \quad \Rightarrow \quad C = 0$

$\Rightarrow \frac{-1}{y} = \frac{-1}{2x^2} \Rightarrow \boxed{y = 2x^2}$

#10. $dy = \frac{\sin x}{\cos x} dx$.

$y = -\ln|\cos x| + C$

$\Uparrow \; y(0) = \frac{3\pi}{2}$

$C = \frac{3\pi}{2}$

$\boxed{y = -\ln|\cos x| + \frac{3\pi}{2}}$