

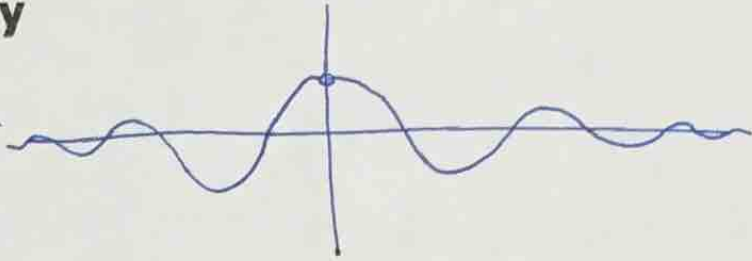
# Trigonometric Limits Notes

## The Big Idea

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

## Graphically

$$f(x) = \frac{\sin x}{x}$$



## Numerically

x	-0.5	-0.1	-0.01	0	0.01	0.1	0.5
$f(x) = \frac{\sin x}{x}$	.959	.998	.999	undef	.999	.998	.959

\*\*Because of this result, all of Calculus is done in RADIANS !!!

## Examples

$$1. \lim_{x \rightarrow \pi} \frac{\sin(x - \pi)}{x - \pi} = 1$$

*SAME and approaching zero*

\* same graph  
but shifted  
right  $\pi$

$$2. \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

↖ not the same ↗

$$\lim_{x \rightarrow 0} \frac{3}{3} \cdot \frac{\sin 3x}{x}$$

$$\lim_{x \rightarrow 0} 3 \cdot \frac{\sin 3x}{3x}$$

↖ SAME! ↗

$$3 \cdot \lim_{x \rightarrow 0} \frac{\sin 3x}{3x}$$

$$3 \cdot 1 = \boxed{3}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin 4x}{7x}$$

$$\lim_{x \rightarrow 0} \frac{1}{7} \cdot \frac{\sin 4x}{x} \cdot \frac{4}{4}$$

$$\frac{4}{7} \cdot \lim_{x \rightarrow 0} \frac{\sin 4x}{4x}$$

$$\frac{4}{7} \cdot 1$$

$$\boxed{\frac{4}{7}}$$

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$$\lim_{x \rightarrow 2} 5x = \left( \lim_{x \rightarrow 2} x \right) \cdot 5$$

$$4. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)}$$

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x} \right)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$1 \cdot \frac{\sin 0}{1 + \cos 0}$$

$$1 \cdot \frac{0}{1 + 1}$$

$$\boxed{0}$$