IB Math HL1 21F and G u-Substitution notes
Objective: Use a change of variables to find an indefinite integral

Warm up: Find indefinite integral by a pattern recognition.

1. $\int e^{5x-3}\,dx$

$= \frac{1}{5} e^{5x-3} + C$

2. $\int (4-3x)^7\,dx$

$= \frac{1}{8}(4-3x)^8 \cdot \frac{1}{-3} + C$

$= -\frac{1}{24}(4-3x)^8 + C$

U-Substitution Method (Use a change of variable)- A pattern recognition method does not work.

1. $\int (x^2+3x)^4 (2x+3)\,dx$

"du.

$u = x^2+3x$.

$\frac{du}{dx} = 2x+3$

$du = (2x+3)dx$

$\int u^4\,du = \frac{1}{5}u^5 + C$

$= \frac{1}{5}(x^2+3x)^5 + C$

2. $\int e^{x^2-x}(2x-1)\,dx$

$u = x^2-x$

$\frac{du}{dx} = 2x-1$

$du = (2x-1)dx$.

$\int e^u\,du = e^u + C$

$= e^{x^2-x} + C$

3. $\int \frac{x^2}{x^3-7}\,dx$

$u = x^3-7$.

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,du$.

$\int \frac{\frac{1}{3}du}{u} = \frac{1}{3}\ln u + C$

$= \frac{1}{3}\ln|x^3-7| + C$

4. $\int \frac{(\ln x)^3}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int u^3\,du = \frac{1}{4}u^4 + C$

$= \frac{1}{4}(\ln x)^4 + C$.

Practice with U-Substitutions) Find the following anti-derivatives.

1. $\int \frac{4x^3}{\sqrt{4x^4+1}}dx$

$u = 4x^4+1$

$du = 16x^3 dx$

$\frac{1}{4}du = 4x^3 dx$

$\int \frac{1}{4} u^{-\frac{1}{2}} du$

$= \frac{1}{2}\sqrt{4x^4+1} + C$

2. $\int (\cos^3\theta \sin\theta)d\theta$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-du = \sin\theta\, d\theta$

$\int -u^3 du$

$= -\frac{1}{4}u^4 + C$

$= -\frac{1}{4}\cos^4\theta + C$

3. $\int (\sec^4\theta \tan\theta)d\theta$

$u = \sec\theta$

$du = \sec\theta \tan\theta\, d\theta$.

$\int u^3 du = \frac{1}{4}\sec^4\theta + C$

4. $\int \frac{6x-9}{(x^2-3x+5)}dx$

$u = x^2-3x+5$

$du = 2x-3\, dx$.

$\int \frac{3\,du}{u} = 3\ln|x^2-3x+5| + C$

5. $\int \frac{\ln(x+1)}{x+1}dx$

$u = \ln x+1$

$du = \frac{1}{(x+1)}dx$

$\int u\, du$

$= \frac{1}{2}[\ln(x+1)]^2 + C$

6. $\int \frac{1}{x(\sqrt[3]{\ln x})}dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int u^{-\frac{1}{3}} du$

$= \frac{3}{2}(\ln x)^{\frac{2}{3}} + C$

7. $\int \tan x\, dx = \int \frac{\sin x}{\cos x}dx$.

$u = \cos x$

$-du = \sin x\, dx$

$\int \frac{-du}{u}$

$= -\ln|\cos x| + C$

OR $\ln|\sec x| + C$

8. $\int \frac{\sin x}{(1+\cos x)^2}dx$

$u = 1+\cos x$

$du = -\sin x\, dx$.

$\int \frac{-du}{u^2} = \frac{1}{1+\cos x} + C$