Definite Integral U-substitution
Objective: Use a change of Variables to evaluate a definite Integral.

Warm UP: Find the indefinite integrals by U-substitution.

1. $\int 2x(x^2+1)^3 dx$

$u = x^2+1$

$du = (2x)du$

$\int u^3 \cdot du$

$= \frac{1}{4}u^4 + c$

$= \boxed{\frac{1}{4}(x^2+1)^4 + c}$

2. $\int 4x^3 \cos(x^4+5) dx$

$u = x^4+5$

$du = 4x^3 du$

$\int \cos u \cdot du$

$= \sin u + c$

$= \sin(x^4+5) + c$

Definite Integral with U-Substitution.

Example 1)

$\int_0^1 2x(x^2+1)^3 dx$

$x=0 \Rightarrow u=1$

$u = x^2+1 \Rightarrow x=1 \Rightarrow u=2$

$du = 2x\,dx$.

$\int_1^2 u^3 du$

$\frac{1}{4}u^4 \Big]_{u=1}^{u=2} = \frac{1}{4}(2)^4 - \frac{1}{4}(1)^4$

$= \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$

$4 - \frac{1}{4}$

Example 2)

$\int_1^3 \frac{x}{\sqrt{3x^2+2}} dx$

$x=1 \Rightarrow u=5$

$u = 3x^2+2$ $\quad x=3 \Rightarrow u=29$

$27+2$

$du = 6x\,dx \Rightarrow \frac{1}{6}du = x\,dx$

$\int_2^{29} \frac{\frac{1}{6}du}{\sqrt{u}}$

$\frac{1}{6}\int_5^{29} u^{-\frac{1}{2}} du$.

$= \frac{1}{6}\left[2u^{\frac{1}{2}}\right]_{u=5}^{u=29}$

$= \frac{1}{3}\left[\sqrt{29} - \sqrt{5}\right]$.

Example 3)

$\int_0^{\sqrt{3}} \frac{\arctan x}{x^2+1} dx \Rightarrow \int_0^{\frac{\pi}{3}} u\,du = \left[\frac{1}{2}u^2\right]_{u=0}^{u=\frac{\pi}{3}}$

$= \frac{1}{2}\left(\frac{\pi}{3}\right)^2$

$u = \arctan x$

$du = \left(\frac{1}{1+x^2}\right)dx$.

$x=0 \Rightarrow u=0, \quad x=\sqrt{3} \Rightarrow \arctan(\sqrt{3}) = \frac{\pi}{3}$

Exit Slip (U-Substitution) : When you completed this Exit Slip, check your answers with your group members. When all your group members agree to the solutions, raise your hands to get a stamp from Mrs. Shim

1. Find the indefinite integral by U-Substitution.

$$\int 2x^2\sqrt{x^3+1}dx \Rightarrow \frac{2}{3}\int u^{\frac{1}{2}}du$$

$du = 3x^2dx$

$\frac{1}{3}du = x^2dx$

$$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C$$

$$= \frac{4}{9}(x^3+1)^{\frac{3}{2}} + C$$

2. Evaluate the definite integral by u-substitution.

$$\int_0^2 2x^2\sqrt{x^3+1}dx = \frac{2}{3}\int_1^9 u^{\frac{1}{2}}du = \left[\frac{4}{9}u^{\frac{3}{2}}\right]_{u=1}^{u=9}$$

$u = x^3+1$

$\frac{1}{3}du = x^2du$

$x=0$

$u=1$

$x=2$

$u=9$

$$= \frac{4}{9}\left[(9)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= \frac{4}{9}\boxed{26} = \boxed{\frac{104}{9}}$$

More U-Substitution (Day Three):

Indefinite Integral:

1. $\int x\sqrt{2x+5}\,dx$

$u = 2x+5 \Rightarrow x = \frac{u-5}{2}$

$du = 2dx$

$\frac{1}{2}du = dx$

$\int \frac{1}{4}(u-5)\sqrt{u}\,du$

$= \frac{1}{4}\int (u^{\frac{3}{2}} - 5u^{\frac{1}{2}})du = \frac{1}{4}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{\frac{3}{2}}\right] + C$

$= \frac{1}{10}(2x+5)^{\frac{5}{2}} - \frac{5}{6}(2x+5)^{\frac{3}{2}} + C$

2. $\int \frac{x}{\sqrt{x+1}}dx \Rightarrow \int \frac{u-1}{\sqrt{u}}du$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$\Rightarrow \int (u-1)\cdot u^{-\frac{1}{2}}du$

$\int (u^{\frac{1}{2}} - u^{-\frac{1}{2}})du = \frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + C$

$= \frac{2}{3}(x+1)^{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} + C$

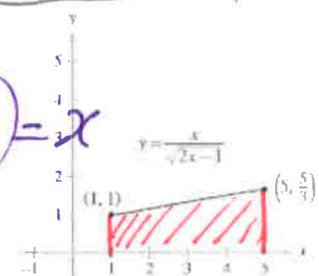
Definite Integral:

1. Evaluate $\int_1^5 \frac{x}{\sqrt{2x-1}}dx$

$u = \sqrt{2x-1} \Rightarrow u^2 = 2x-1 \Rightarrow \frac{u^2+1}{2} = x$

$du = (\tfrac{1}{2})(2)(2x-1)^{-\frac{1}{2}}dx$

$du = \frac{1}{\sqrt{2x-1}}dx$



$\Delta x \approx dx$

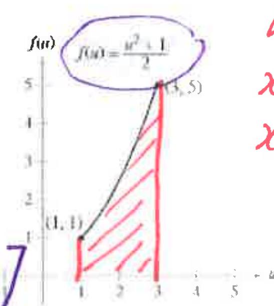
The region before substitution has an area of $\frac{16}{3}$.

Figure 4.38

$u = \sqrt{2x-1}$

$x = 1 \Rightarrow u = 1$

$x = 5 \Rightarrow u = 3$



$\Delta u \approx du$

The region after substitution has an area of $\frac{16}{3}$.

Figure 4.39

$\frac{1}{2}\int_1^3 (u^2+1)du$

$= \frac{1}{2}\left[\frac{u^3}{3} + u\right]_{u=1}^{u=3} = \frac{1}{2}\left[9+3-\frac{1}{3}-1\right]$

$\frac{33}{3} - \frac{1}{3}$ $\quad \frac{32}{3}$

$= \frac{16}{3}$

2. Evaluate $\int_1^3 x\sqrt{2x+5}\,dx$

$x = 2x+5$ $\quad x=1 \quad u = 7$

$x = 3 \quad u = 11$

$\frac{1}{4}\int_7^{11} (u^{\frac{3}{2}} - 5u^{\frac{1}{2}})du = \left[\frac{1}{10}(u)^{\frac{5}{2}} - \frac{5}{6}(u)^{\frac{3}{2}}\right]_7^{11}$

$= \left[\frac{1}{10}(11)^{\frac{5}{2}} - \frac{5}{6}(11)^{\frac{3}{2}}\right] - \left[\frac{1}{10}(7)^{\frac{5}{2}} - \frac{5}{6}(7)^{\frac{3}{2}}\right]$

12.2 unit