Integration by partial fraction WS. Answers.

a. $\int \frac{x+4}{(x-1)(x+6)}\,dx = \int \frac{A}{x-1}\,dx + \int \frac{B}{x+6}\,dx = \frac{5}{7}\ln|x-1| + \frac{2}{7}\ln|x+6| + C$

$A(x+6) + B(x-1) = x+4$

OR $\ln \frac{(x-1)^{5/7}}{(x+6)^{2/7}}$

$x=1 \quad A = \frac{5}{7} \quad B = \frac{2}{7}.$

$x=-6 \quad B = \frac{-2}{-7}$

b. $\int \frac{1}{x^3+x^2-2x}\,dx = \int \frac{A}{x}\,dx + \int \frac{B}{(x+2)}\,dx + \int \frac{C}{(x-1)}\,dx$

$A(x+2)(x-1) + Bx(x-1) + Cx(x+2) = 1$

$\int -\frac{1}{2}\cdot\frac{1}{(x)}\,dx + \int \frac{1}{6}\frac{1}{(x+2)}\,dx + \int \frac{1}{3}\frac{1}{(x-1)}\,dx$

$x=1 \quad C = \frac{1}{3}$

$= -\frac{1}{2}\ln|x| + \frac{1}{6}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$

$x=-2 \quad B(-2)(-3) = 1 \quad B = \frac{1}{6}$

$x=0 \quad A(2)(-1) = 1 \quad A = \frac{-1}{2}.$

c. $\int \frac{2x+1}{x^2-7x+12}\,dx = \int \frac{A}{(x-3)}\,dx + \int \frac{B}{(x-4)}\,dx = -7\ln|x-3| + 9\ln|x-4| + C$

$A(x-4) + B(x-3) = 2x+1$

$= \ln \frac{(x-4)^9}{(x-3)^7} + C$

$x=4 \quad B = 9$

$x=3 \quad -A = 7 \quad A = -7$

d. $\int \frac{2x-1}{(x-1)^2}\,dx = \int \frac{A}{x-1}\,dx + \int \frac{B}{(x-1)^2}\,dx = \int \frac{2}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$

$= 2\ln|x-1| - \frac{1}{x-1} + C$

$A(x-1) + B = 2x-1$

$x=1 \quad B = 1$

$x=0 \quad -A+1 = -1 \quad A = 2$

e. $\displaystyle\int \frac{1}{(x+1)(x^2+1)}\,dx = \int\left[\frac{A}{x+1} + \frac{Bx+C}{(x^2+1)}\right]dx$

$\Rightarrow A(x^2+1) + (Bx+C)(x+1) = 1$

$x=-1 \quad 2A = 1 \quad A = \frac{1}{2}$

$x=0 \quad \frac{1}{2} + C = 1 \quad C = \frac{1}{2}$

$x=1 \quad (\frac{1}{2})(2) + (B+\frac{1}{2})(2) = 1$

$2B+1=0 \quad B = \frac{-1}{2}$

$\displaystyle\Rightarrow \int \frac{\frac{1}{2}}{x+1}\,dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1}\,dx$

$= \frac{1}{2}\ln|x+1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}x + C$

f.