

$$f. \int \frac{x^2+3x+1}{x^3-1} dx.$$

$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$= \int \frac{x^2+3x+1}{(x-1)(x^2+x+1)} dx$$

$$= \int \frac{A}{x-1} dx + \int \frac{Bx+C}{x^2+x+1} dx$$

$$\Rightarrow \underline{A(x^2+x+1) + (Bx+C)(x-1) = x^2+3x+1}$$

• $x=1$ $A(1+1+1) = 1+3+1$ $A = \frac{5}{3}$

• $x=0$ $\frac{5}{3}(0+0+1) + (C)(-1) = 1$ $C = \frac{2}{3}$

• $x=-1$ $\frac{5}{3}(1-1+1) + (-B + \frac{2}{3})(-2) = 1-3+1$
 $\frac{5}{3} + (2B) - \frac{4}{3} = -1$ $B = -\frac{2}{3}$

$$\rightarrow \int \frac{\frac{5}{3}}{x-1} dx + \int \frac{-\frac{2}{3}x + \frac{2}{3}}{x^2+x+1} dx.$$

$$= \frac{5}{3} \ln|x-1| + \frac{1}{3} \int \frac{-2x+2}{x^2+x+1} dx$$

$$u = x^2+x+1$$

$$du = 2x+1$$

$$= // + \frac{1}{3} \int \frac{-(2x+1) + 3}{x^2+x+1}$$

$$= // + \frac{-1}{3} \int \frac{2x+1}{x^2+x+1} dx + \int \frac{1}{x^2+x+1} dx$$

$$= // - \frac{1}{3} \ln|x^2+x+1| + \int \frac{1}{x^2+x+\frac{1}{4} + \frac{3}{4}} dx.$$

$$= \dots + \dots + \int \frac{1}{(x + \frac{1}{2})^2 + \frac{3}{4}} dx \quad \begin{matrix} u = x + \frac{1}{2} \\ du = dx \end{matrix}$$

$$= \dots + \dots + \int \frac{1}{u^2 + (\frac{\sqrt{3}}{2})^2} du$$

$$= \frac{5}{3} \ln|x-1| - \frac{1}{3} \ln|x^2+x+1| + \frac{2}{\sqrt{3}} \arctan\left(\frac{u}{\frac{\sqrt{3}}{2}}\right) + C$$

$$= \left[\frac{5}{3} \ln|x-1| - \frac{1}{3} \ln|x^2+x+1| + \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C \right]$$