IB Math HL1 Indefinite Integral Extra Practice (Mixed) Key ①

Warm-Up: Integrate

$\int 5x\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int \frac{-5}{2} u^{\frac{1}{2}} du$

$= \frac{-5}{2} \cdot \frac{2}{3} (1-x^2)^{3/2} + C$

$= \boxed{-\frac{5}{3}(1-x^2)^{3/2} + C}$

2. $\int \frac{\sec^2 x}{(\tan x - 7)^3} dx$

$u = \tan x - 7$

$du = \sec^2 x\,dx$

$\int \frac{du}{u^3}$

$= \boxed{\frac{-1}{2(\tan x - 7)^2} + C}$

Practice)

1. $\int \frac{4}{x-7} dx$

$= \boxed{4 \ln|x-7| + C}$

2. $\int \frac{4x}{x^2+5} dx$

$u = x^2+5$

$du = 2x\,dx$

$\int \frac{2 \cdot du}{u}$

$= \boxed{2 \ln|x^2+5| + C}$

3. $\int \frac{2x-5}{4x^2} dx$

$= \int \left(\frac{2x}{4x^2} - \frac{5}{4x^2}\right) dx$

$= \int \left(\frac{1}{2}x^{-1} - \frac{5}{4}x^{-2}\right) dx = \boxed{\frac{1}{2}\ln|x| + \frac{5}{4x} + C}$

4. $\int \frac{6x-9}{(x^2-3x+5)^3} dx = \int \frac{3\,du}{u^3}$

$u = x^2 - 3x + 5$

$du = (2x-3)dx$

$= \boxed{\frac{-3}{2(x^2-3x+5)^2} + C}$

5. $\int \frac{x^3 + \sqrt[5]{x^3}}{\sqrt[5]{x^7}} dx = \int \frac{x^3}{x^{7/5}} + \frac{x^{3/5}}{x^{7/5}}$

$= \int \left(x^{8/5} + x^{-4/5}\right) dx$

$= \boxed{\frac{5}{13}x^{13/5} + 5x^{1/5} + C}$

6. $\int (\tan^2 x)\,dx$

$\cos^2 x + \sin^2 x = 1$

$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\boxed{1 + \tan^2 x = \sec^2 x}$

$= \int (\sec^2 x - 1)\,dx$

$= \boxed{\tan x - x + C}$

7. $\int \cos x(e^{\sin x})dx = \boxed{e^{\sin x} + C}$

$u = \sin x$

$du = \cos x\, dx$

8. $\int \left(x+\sqrt{x}\right)^2 dx = \int (x^2 + 2x^{3/2} + x)dx$

$= \frac{1}{3}x^3 + 2\cdot\frac{2}{5}x^{5/2} + \frac{1}{2}x^2 + C$

$= \boxed{\frac{1}{3}x^3 + \frac{4}{5}x^{5/2} + \frac{1}{2}x^2 + C}$

9. $\int \frac{2}{x}(\ln x)^2 dx = \int 2u^2 du$

$= \boxed{\frac{2}{3}(\ln x)^3 + C}$

$u = \ln x$

$du = \frac{1}{x}dx$

10. $\int xe^{1-x^2} dx = \int -\frac{1}{2} e^u du$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \boxed{-\frac{1}{2} e^{1-x^2} + C}$

11. $\int \frac{x+4}{\sqrt{4-x^2}}dx = \int \frac{x}{\sqrt{4-x^2}}dx + \int \frac{4}{\sqrt{4-x^2}}dx$

$= \boxed{-\sqrt{4-x^2} + 4 \arcsin\left(\frac{x}{2}\right) + C}$

The details of work attached.

12. $\int \frac{x^2}{x^2+1}dx = \int \left(\frac{x^2+1-1}{x^2+1}\right)dx$

$= \int \left(1 - \frac{1}{x^2+1}\right)dx$

$= \boxed{1 - \arctan x + C}$

13. $K''(x) = \cos x - 5e^x$, $K'(0) = 2$, and $K(0) = 5$. Find $K(x)$

$K'(x) = \int (\cos x - 5e^x)dx$

$= \sin x - 5e^x + C$

$K'(0) = \sin 0 - 5e^0 + C = 2$

$C = 7$

$\Rightarrow K(x) = \int (\sin x - 5e^x + 7)dx$

$K(x) = -\cos x - 5e^x + 7x + C_2$

$K(0) = -\cos(0) - 5\cdot e^0 + 7\cdot 0 + C_2 = 5$

$= -1 - 5 + C_2 = 5 \quad C_2 = 11$

$\Rightarrow \boxed{K(x) = -\cos x - 5e^x + 7x + 11}$

#11. $\int \frac{x+4}{\sqrt{4-x^2}}\,dx$

$\Rightarrow \int \frac{x}{\sqrt{4-x^2}} + \frac{4}{\sqrt{4-x^2}}\,dx$

① ②

① $u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int -\frac{1}{2} u^{-\frac{1}{2}}\,du$

$= -\frac{1}{2}\cdot 2\, u^{\frac{1}{2}} + C$

$= -\sqrt{4-x^2} + C$

$\int \frac{4}{\sqrt{4-x^2}}\,dx$

$= \int \frac{4}{\sqrt{4}\sqrt{1-(\frac{x}{2})^2}}\,dx$

$= \int \frac{2}{\sqrt{1-(\frac{x}{2})^2}}\,dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$dx = 2\,du.$

$\int \frac{4du}{\sqrt{1-u^2}}$

$= 4\arcsin u$

$= 4\arcsin\left(\frac{x}{2}\right) + C$

$\Rightarrow -\sqrt{4-x^2} + 4\arcsin\left(\frac{x}{2}\right) + C$