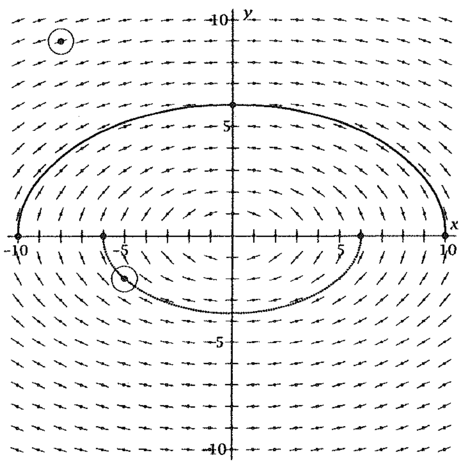
Key for the Slope field W.S Answers.

Exploration 7-4a

1. $\left.\frac{dy}{dx}\right|_{(-5,\,-2)} = -0.9$ $\left.\frac{dy}{dx}\right|_{(-8,\,9)} = 0.32$
   The graph shows that the slopes at these points look reasonably close to −0.9 and 0.32.



2. See the solid line in the graph in Problem 1. The graph looks like a half-ellipse. Below the x-axis, the graph would complete the ellipse, but would not satisfy the definition of a solution of a differential equation because it would not be a function.

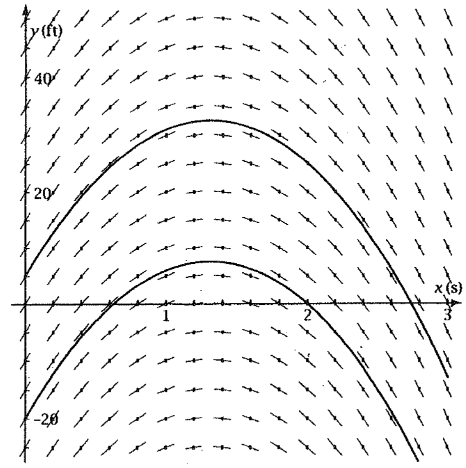
3. See the dashed line in the graph in Problem 1. The graph looks like another half-ellipse of the same proportions, but smaller than the half-ellipse in Problem 2. This time only the bottom half of the ellipse is valid, because the initial y-value is negative.

4. $\frac{dy}{dx} = -\frac{0.36x}{y} \Rightarrow y\,dy = -0.36x\,dx$
   $\int y\,dy = \int -0.36x\,dx$
   $\frac{1}{2}y^2 = -\frac{1}{2} \cdot 0.36x^2 + C$
   $\left(\frac{x}{10}\right)^2 + \left(\frac{y}{6}\right)^2 = C_1$
   This is a standard form of the equation of an ellipse centered at the origin with x- and y-radii equal to $10\sqrt{C_1}$ and $6\sqrt{C_1}$, respectively.

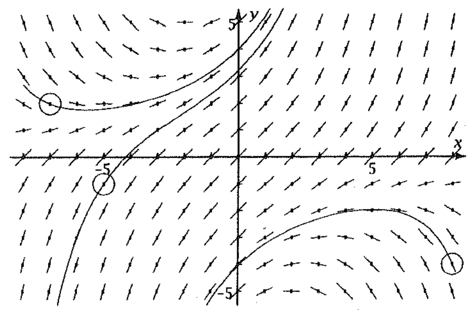
5. Answers will vary.

Exploration 7-4b

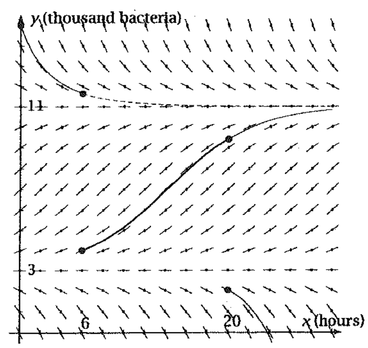
1. See the graph with initial condition (0, 5). The maximum height is at $x \approx 1.3$ s, and the ball hits the ground (height = 0) at $x \approx 2.7$ s. See the graph with initial condition (0, −20). The ball hits the ground ($y = 0$) when $x \approx 0.6$ s or 2.0 s.



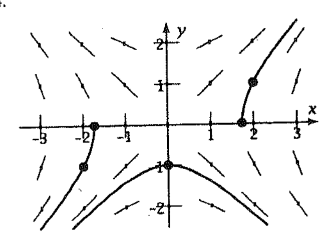
2. The first solution reaches a minimum at $x \approx -6$. The second solution rises at a decreasing rate, then at an increasing rate, approaching the first solution along a curved asymptote. The third solution reaches a maximum at $x \approx 5$.



3. Without any treatments, the number of bacteria would decrease to an asymptote at $y = 11$, the maximum sustainable population. After the first treatment, the number of bacteria *rises* toward the same asymptote (because $y = 4$ is below the maximum sustainable population but above the minimum). After the second treatment, the bacteria decrease and become extinct (because 2 is below the minimum sustainable population).



4.

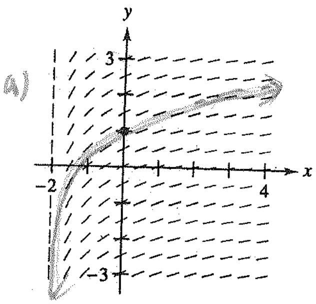


IB Math 2: More Practice of Slope Field: Name: Key Period: ______

In these exercises, a differential equation, a point, and a slope field are given.
(a) Sketch two approximate solutions, one of which passes through the given point.
(b) Use integration to find the particular solution of the differential equation.
(c) Use graphing utility to graph the solutions from (a) and (b), and sketch the graphs to compare the results.

1. $\dfrac{dy}{dx} = \dfrac{1}{x+2}$, $(0, 1)$

a)



$dy = \left(\frac{1}{x+2}\right)dx$

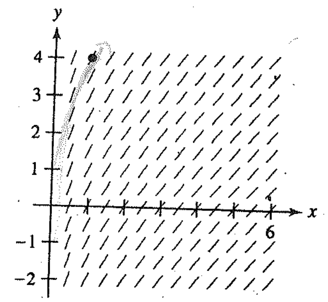
$y = \ln|x+2| + C.$

$1 = \ln 2 + C$

$C = 1 - \ln 2.$

$y = \ln|x+2| + (1 - \ln 2)$

2. $\dfrac{dy}{dx} = 1 + \dfrac{1}{x}$, $(1, 4)$



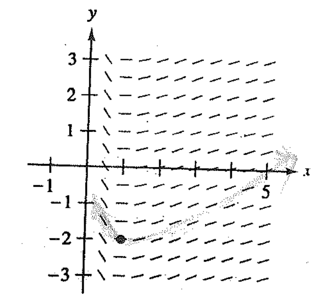
$dy = (1 + \frac{1}{x})dx$

$y = x + \ln x + C$

$4 = 1 + C$ $\boxed{C = 3}$

$y = (x + \ln x + c)$

3. $\dfrac{dy}{dx} = \dfrac{\ln x}{x}$, $(1, -2)$



$dy = \left(\frac{\ln x}{x}\right)dx.$

$u = \ln x \quad du = \frac{1}{x}dx$

$dy = u \cdot du$

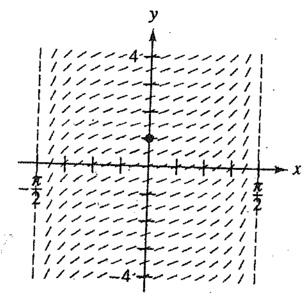
$y = \frac{1}{2}(\ln x)^2 + C$

$-2 = \frac{1}{2}(0)^2 + C$

$C = -2$

$\Rightarrow y = \frac{1}{2}(\ln x)^2 - 2$

4. $\dfrac{dy}{dx} = \sec x$, $(0, 1)$



$dy = \sec x\, dx$

$dy = \dfrac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)}dx$

$dy = \dfrac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}dx$

$u = \sec x + \tan x$ $\int dy = \int \frac{1}{u}du.$

$du = (\sec^2 x + \sec x \tan x)dx$

$y = \ln|\sec x + \tan x| + C$

$C = 1$

$\Rightarrow y = \ln|\sec x + \tan x| + C$