14K.2 Geometric Properties of the Cross Product

Given: $A(1,4,2)$, $B(0,8,0)$, and $C(4,10,3)$

1. Find $m\hat{ABC}$ in radians.
2. Find the area of ΔABC.
3. Find a vector **v** which is perpendicular to $\overrightarrow{BA}$ and $\overrightarrow{BC}$.
4. Find $|\mathbf{v}|$.
5. Compare the results of #2 and #4.

1. $\vec{BA} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix} \qquad \vec{BC} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$

$$\vec{BA} \cdot \vec{BC} = |\vec{BA}|\,|\vec{BC}| \cos\theta$$

$$4 - 8 + 6 = \sqrt{21}\sqrt{29} \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{21}\sqrt{29}}\right) \approx \underline{1.49 \text{ radians}}$$

2. $\text{Area} = \frac{1}{2}|\vec{BA}|\,|\vec{BC}| \sin\theta$

$$= \frac{1}{2}\sqrt{21}\sqrt{29} \sin 1.49 \approx \underline{12.3}$$

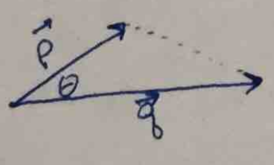
3. $\vec{v} = \vec{BA} \times \vec{BC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = \begin{pmatrix} -12-4 \\ -(3-8) \\ 2+16 \end{pmatrix} = \begin{pmatrix} -16 \\ 5 \\ 18 \end{pmatrix}$$

4. $|\vec{v}| = \sqrt{605} \approx 24.6$

5. $|\vec{BA} \times \vec{BC}| = 2\,(\text{Area of } \Delta ABC)$

$$|\vec{p} \times \vec{q}| = 2\,(\text{Area of triangle})$$



$$|\vec{p} \times \vec{q}| = 2\left(\tfrac{1}{2}|\vec{p}|\,|\vec{q}| \sin\theta\right)$$

$$\boxed{|\vec{p} \times \vec{q}| = |\vec{p}|\,|\vec{q}| \sin\theta}$$