

14K.2 Geometric Properties of the Cross Product

Round answers to 3 decimal places.

Given $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, find:

1. $\mathbf{a} \cdot \mathbf{b}$

$$\begin{aligned} & (3)(-2) + (2)(1) + (1)(2) \\ & -6 + 2 + 2 \\ & \boxed{-2} \end{aligned}$$

2. θ , the angle between \mathbf{a} and \mathbf{b} .

$$-2 = \sqrt{14} \sqrt{9} \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{-2}{3\sqrt{14}} \right)$$

$$\boxed{\theta \approx 100.263^\circ}$$

3. $\mathbf{a} \times \mathbf{b}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

$$\begin{aligned} & (4-1)\hat{i} - (6+2)\hat{j} + (3+4)\hat{k} \\ & \boxed{3\hat{i} - 8\hat{j} + 7\hat{k}} \end{aligned}$$

4. $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 7 \end{pmatrix}$$

$$9 - 16 + 7$$

$$\boxed{0}$$

5. $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{b})$

$$\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 7 \end{pmatrix}$$

$$-6 - 8 + 14$$

$$\boxed{0}$$

6. $|\mathbf{a} \times \mathbf{b}|$

$$\sqrt{9 + 64 + 49}$$

$$\sqrt{122} \approx \boxed{11.045}$$

7. $|\mathbf{a}||\mathbf{b}|\sin \theta$

$$\sqrt{14} \cdot 3 \sin 100.263^\circ$$

$$\approx \boxed{11.045}$$

Direction of $\mathbf{a} \times \mathbf{b}$: $\mathbf{a} \times \mathbf{b}$ is perpendicular to \mathbf{a} and \mathbf{b} .

Length of $\mathbf{a} \times \mathbf{b}$: $|\mathbf{a} \times \mathbf{b}| = \underline{|\mathbf{a}||\mathbf{b}|\sin \theta}$

Key