IB Questions with Calculator

#1. a) $x_A \approx 2.87$

$x_B \approx 6.78$

b) $\displaystyle\int_{2.87}^{6.78} (1 - 2\sin x - x^2 e^{-x})\,dx \approx \boxed{6.76}$.

#2. $(x^2+y^2)^2 = 4xy^2$

$x^4 + 2x^2y^2 + y^4 = 4xy^2$

$4x^3 + 4xy^2 + 4x^2y \cdot y' + 4y^3y' = 4y^2 + 8xyy'$

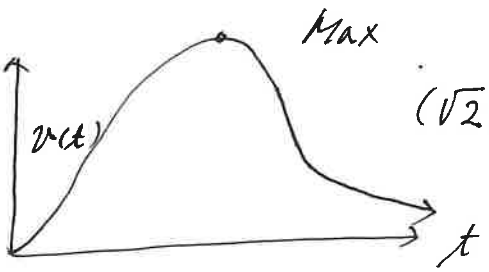
a) $\dfrac{dy}{dx} = \dfrac{-x^3 - xy^2 + y^2}{yx^2 - 2xy + y^3}$

b) $\dfrac{dy}{dx}(1,1) = \dfrac{-1}{0}$ Undefined. $\Rightarrow$ Normal line.

$\boxed{y = 1}$

#3.

a)



$\left(\sqrt{2}, \dfrac{\sqrt{2}}{16}\right)$ OR $(1.41, 0.0884)$

OR

$\dfrac{dv}{dt} = \dfrac{(12+t^4) - (t)(4t^3)}{(12+t^4)^2} \Rightarrow 12 - 3t^4 = 0$

$t^4 = 4 \quad t = \sqrt{2}$.

$v(\sqrt{2}) = \dfrac{\sqrt{2}}{16}$

$\underset{0}{|} \;\oplus\; \underset{\sqrt{2}}{|} \;\ominus$ Sign diagram of $v'(t)$

b) $u = t^2$

$du = 2t \cdot dt$

$dt = \frac{du}{2t}$

$$\int \frac{\cancel{t}}{12 + u^2} \cdot \frac{du}{2\cancel{t}} = \frac{1}{2} \int \frac{du}{u^2 + 12}$$

$$= \frac{1}{2\sqrt{12}} \arctan\left(\frac{u}{\sqrt{12}}\right) + C$$

$$= \frac{1}{2\sqrt{12}} \arctan\left(\frac{t^2}{\sqrt{12}}\right) + C .$$

c) $$\int_0^6 \frac{t}{12 + t^4}\, dt = \int_0^{36} \frac{du}{u^2 + 12} = \frac{1}{2\sqrt{12}} \left[\arctan\left(\frac{36}{\sqrt{12}}\right) - 0\right]$$

$t = 0 \Rightarrow u = 0$

$t = 6 \Rightarrow u = 36$

$$= \frac{\arctan(3\sqrt{12})}{2\sqrt{12}} \text{ OR } \frac{\arctan(6\sqrt{3})}{4\sqrt{3}}$$

d) $$v(s) = \arcsin(\sqrt{s}) \Rightarrow \frac{dv}{ds} = \frac{1}{2\sqrt{s(1-s)}}$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v$$

$$= \arcsin(\sqrt{s}) \cdot \frac{1}{2\sqrt{s(1-s)}}$$

$$a_{(s=0.1m)} = \arcsin(\sqrt{0.1}) \cdot \frac{1}{2\sqrt{0.1(1-0.1)}} \approx 0.536 \ m/s^2$$