Integration with Trig Identities (Day three).

$\sin^2 x+\cos^2 x=1$	$\tan^2 x+1=\sec^2 x$	$\cos^2 x=\frac{1}{2}(1+\cos 2x)$	$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

1. **Powers of sine and cosine.**

Example 1) Evaluate $\int \sin^3 x\cos x\,dx$

$u=\sin x$

$du=\cos x\,dx$

$\int u^3\,du = \frac{1}{4}\sin^4 x+C$

Example 2) Evaluate $\int \cos^2 x\,dx$

$\int \frac{1}{2}(1+\cos 2x)\,dx$

$=\frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)+C$

2. **Powers of Secant and tangent.**

Example 3) Evaluate $\int \sec^6 x\tan x\,dx$

$\int \sec^5 x\,(\sec x\cdot\tan x)\,dx$

$u=\sec x$

$du=(\sec x)(\tan x)\,dx$

$\int u^5\,du = \frac{1}{6}\sec^6 x+C$

Example 3) Evaluate $\int \sec^2 x\tan^2 x\,dx$

$u=\tan x$

$du=\sec^2 x\,dx$

$\int u^2\,du$

$=\frac{1}{3}\tan^3 x+C$

Practice) ⇒ Solutions attached

1. $\int \cos^3 x\,dx = \int \cos^2 x\cdot\cos x\,dx$
   $= \int (1-\sin^2 x)\cos x\,dx$
2. $\int e^{\cos x}\sin x\,dx$
3. $\int \tan 2\theta\,d\theta$
4. $\int \frac{\cos x}{1+3\sin x}\,dx$
5. $\int e^x\sin(e^x)\,dx$
6. $\int \sqrt[3]{\tan x}\sec^2 x\,dx$
7. $\int \sin^2 x\,dx$

Example 4) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$

$u = \cos x \qquad du = -\sin x \, dx$

$\int -\frac{1}{u} \, du = -\ln|\cos x| + C$

$= \ln|\sec x| + C$

Example 5) $\int \frac{\sec x}{1} \, dx$ $(\sec x + \tan x)$ $(\sec x + \tan x)$

$\int \sqrt{1 + \tan^2 x} \, dx$

$\int \frac{\sec^2 x + \sec x \cdot \tan x}{(\sec x + \tan x)} \, dx = \int \frac{du}{u}$

$= \ln|\sec x + \tan x| + C$

$u = \sec x + \tan x$

$du = (\sec x \cdot \tan x + \sec^2 x) \, dx$

practice)

#1 $\int \cos^3 x\,dx = \int (1-\sin^2 x)\cos x\,dx$.

$\begin{pmatrix} u = \cancel{\cos}\ \sin x. \\ du = \cos x\,dx \end{pmatrix} \rightarrow \int (1-u^2)\,du$

$= u - \frac{1}{3}u^3 + C = \sin x - \frac{1}{3}\sin^3 x + C$

#2. $\int \tan 2\theta\,d\theta = \int \frac{\sin 2\theta}{\cos 2\theta}\,d\theta$

$\begin{pmatrix} u = \cos 2\theta \\ -\frac{1}{2}du = \sin 2\theta\,d\theta \end{pmatrix} = -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|\cos 2\theta| + C$

#3. $\int e^{\cos x}\sin x\,dx = -e^{\cos x} + C$

#4 $\int \frac{\cos x}{1+3\sin x}\,dx = \frac{1}{3}\ln|1+3\sin x| + C$

$\begin{pmatrix} u = 1+3\sin x \\ \frac{1}{3}du = \cos x\,dx \end{pmatrix}$.

#5. $\int e^x \sin(e^x)\,dx = -\cos(e^x) + C$

#6. $\int u^{\frac{1}{3}}\,du = \frac{3}{4}\tan^{\frac{4}{3}} x + C$

$\begin{pmatrix} u = \tan x \\ du = \sec^2 x\,dx \end{pmatrix}$

#7. $\int \sin^2 x\,dx = \frac{1}{2}\int (1-\cos 2x)\,dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$