IB Math HL1 Exit Slip #3 **Name:** key

Find the indefinite integral.

1. $\int (t - t\sqrt{t})dt$ $= \int t - t^{\frac{3}{2}}$

$= \frac{1}{2}t^2 - \frac{2}{5}t^{5/2} + c$

$= \frac{1}{2}t^2 - \frac{2}{5}t^2\sqrt{t} + c$

Answer: ____________

2. $\int (\cot x)dx$ $= \int \frac{\cos x}{\sin x}dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int \frac{du}{u}$

$= \ln|\sin x| + c$

Answer: ____________

3. $\int (e^{-5x} - \frac{2}{x})dx$

$= -\frac{1}{5}e^{-5x} - 2\ln|x| + c$

Answer: ____________

4. $\int \frac{x^3}{\sqrt[4]{1+2x^4}}dx$ $= \frac{1}{8}\int \frac{du}{\sqrt[4]{u}} = \frac{1}{8}\int u^{-\frac{1}{4}}du$

$u = 1 + 2x^4$

$du = 8x^3dx \rightarrow x^3dx = \frac{1}{8}du$

$= \frac{1}{8}\cdot\frac{4}{3}(1+2x^4)^{\frac{3}{4}} + c$

$= \frac{1}{6}(1+2x^4)^{3/4} + c$

Answer: $\frac{1}{6}(1+2x^4)^{3/4} + c$

5. $\int \frac{\sin 3x}{5 + \cos 3x}dx$ $= -\frac{1}{3}\ln|5 + \cos 3x| + c$

$u = 5 + \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin(3x)dx$

Answer: ____________

6. $\int \frac{2e^{(\sqrt{x}-3)}}{\sqrt{x}}dx$ $\rightarrow \int 4e^u du = 4e^{\sqrt{x}-3} + c$

$u = \sqrt{x} - 3$

$du = \frac{1}{2}x^{-\frac{1}{2}}dx \Rightarrow 2du = x^{-\frac{1}{2}}dx$

Answer: ____________

7. Evaluate $\int_0^{\sqrt{3}} \frac{3x^2+5}{x^2+1}dx$ (exact answer only)

$3x^2 + 3 + 2$

$\int_0^{\sqrt{3}} \left(\frac{3(x^2+1)}{x^2+1} + \frac{2}{x^2+1}\right)dx$

$= \int_0^{\sqrt{3}} 3dx + 2\int_0^{\sqrt{3}} \frac{1}{x^2+1}dx$

Answer: $= 3x\Big|_0^{\sqrt{3}} + 2\arctan x\Big|_{x=0}^{\sqrt{3}}$

$= 3\sqrt{3} + 2[\arctan\sqrt{3} - \arctan 0]$

$= 3\sqrt{3} + 2\left(\frac{\pi}{3}\right)$

8. $\int \frac{5x-1}{x-3}dx$

(hint: write In the forms of $\int A + \frac{B}{x-3}dx$

$\int \left(\frac{5x-1}{x-3}\right)dx = \int\left(\frac{5(x-3)}{x-3} + \frac{14}{x-3}\right)dx$

Answer: ____________ $= \int 5dx + \int \frac{14}{x-3}dx$

$= 5x + 14\ln|x-3| + c$

9. Evaluate $\int_0^4 7x(x-2)^5\,dx$ (use substitution)

$u = x-2 \Rightarrow x = u+2$

$du = dx$

$x=0 \quad u=-2$

$x=4 \quad u=2$

$\int_{-2}^{2} 7(u+2)u^5\,du = 7\int_{-2}^{2}(u^6+2u^5)\,du$

$\rightarrow 7\left[\frac{1}{7}u^7+\frac{2}{6}u^6\right]_{-2}^{2}$

$= 7\left[\frac{2^7}{7}+\frac{2}{6}2^6-\frac{(-2)^7}{7}-\frac{2}{6}(-2)^6\right]$

$= \boxed{2(2)^7} = \boxed{2^8}$

Suppose f and h are continuous and that

$\int_1^7 f(x)dx = -1 \qquad \int_7^9 f(x)dx = 5 \qquad \int_1^9 g(x)dx = \frac{3}{2} \qquad \int_7^9 g(x)dx = 4$

$\int_1^9 f(x) = (-1)+5 = 4 \Rightarrow \int_9^1 f(x) = -4$

$\int_1^7 g(x) = \int_1^9 g(x)dx - \int_7^9 g(x)dx = \frac{3}{2}-4 = \frac{3}{2}-\frac{8}{2} = \frac{-5}{2}$

10. $\int_9^1 2f(x)dx =$ $2(-4) = \boxed{-8}$

11. $\int_1^7 [f(x)-3g(x)]dx =$ $(-1)-3\left(-\frac{5}{2}\right) = -1+\frac{15}{2} = \frac{13}{2}$

12. Given $\int \frac{5}{x(\ln^2 x - 2\ln x + 5)}dx$,

a) Rewrite $(\ln^2 x - 2\ln x + 5) = (\ln x + B)^2 + C$

$= (\ln x - 1)^2 + 4$

b) Hence, find $\int \frac{5}{x(\ln^2 x - 2\ln x + 5)}dx$

$u = \ln x - 1$

$du = \frac{1}{x}dx$

$= \int \frac{5}{(u)^2+4}du = 5\left(\frac{1}{2}\right)\arctan\left(\frac{\ln x - 1}{2}\right) + C \Rightarrow \boxed{\frac{5}{2}\arctan\left(\frac{\ln x - 1}{2}\right) + C}$

13. Given $f(x) = -5x^2 + 10$,

a) Write in sigma notion of the estimated area under the curve by 10 right hand rectangles in [1, 3].

$\Delta x = \frac{3-1}{10} = \frac{1}{5} \Rightarrow \sum_{k=1}^{10} \frac{1}{5}\left[-5\left[1+\frac{1}{5}k\right]^2+10\right] \approx -27.4$

$-5x^2+10=0$

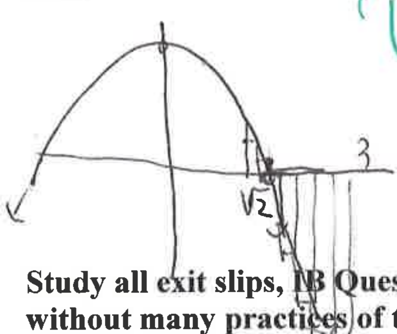
$x^2 = \frac{-10}{-5} = 2$

$x = \sqrt{2}$

b) Hence, discuss if the estimated area is overestimated or underestimated comparing with the area found by FTC.

$-27.4 < -23.\overline{3}$

underestimate (In term of Quantity ⇒ Over Estimate)



by FTC $\int_1^3 (-5x^2+10)dx = -5\left[\frac{x^3}{3}+10x\right]_1^3$

$= -23.\overline{3}$

Study all exit slips, IB Questions, Quizzes, and the review questions. Do not expect you will do well without many practices of these problems.