More practice problems #1 ~ #10. key. P

#1. line $\frac{x-3}{4} = \frac{y-1}{2} = \frac{z+2}{3}$ plane $2x - y - 2z = 0$

$x = 3 + 4t$
$y = 1 + 2t$
$z = -2 + 3t$

$\Rightarrow 2(3+4t) - (1+2t) - 2(-2+3t) = 0$

$6 + 8t - 1 - 2t + 4 - 6t = 0$

$9 \neq 0$ No solution

The line and plane do not intersect.

$(4i + 2j + 3k) \cdot (2i - j - 2k) = 8 - 2 - 6 = 0 \Rightarrow$ They are parallel.

#2. a) parallel $k = -6$.

$3x + 2y - z = 5$
$\cdot(x-2)$ $kx - 4y + 2z = -2$

$(3i + 2j - k) \cdot (ki - 4j + 2k) = 0$

b) $3k - 8 - 2 = 0$ $k = \boxed{\frac{10}{3}}$

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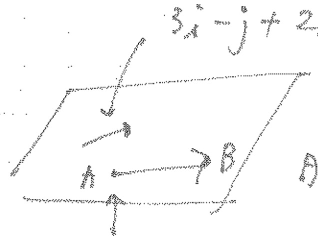
#4.



$\vec{AB} = \begin{pmatrix} 1-5 \\ 0-2 \\ 4+1 \end{pmatrix} = -4i - 2j + 5k$.

A (5, 2, -1) $\Rightarrow \vec{r} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}$

vector Equation.

OR.

Cartesian Equation... $\begin{vmatrix} i & j & k \\ 3 & -1 & 2 \\ -4 & -2 & 5 \end{vmatrix} = i(-5+4) - j(15+8) + k(-6-4)$

$= -i - 23j - 10k$.

$\Rightarrow i + 23j + 10k$

B (1, 0, 4)

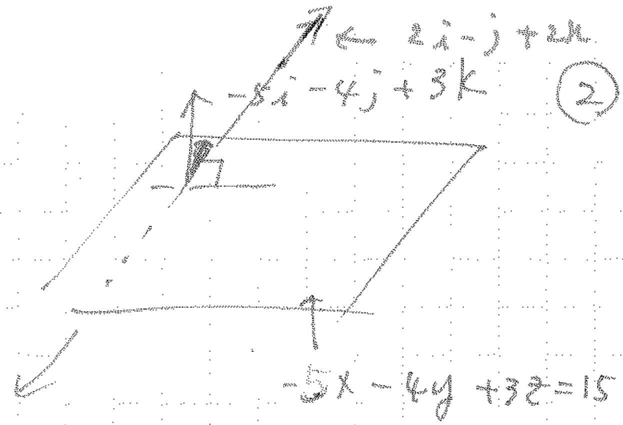
$\Rightarrow x + 23y + 10z = D$

$1 + 40 = D = 41$

$\Rightarrow \boxed{x + 23y + 10z = 41}$

②

#3. line $\begin{cases} x = 3 + 2t \\ y = 4 - t \\ z = 1 - 2t \end{cases}$



$$\cos\theta = \frac{(2i - j - 2k) \cdot (-5i - 4j + 3k)}{\sqrt{4+1+4}\ \sqrt{25+16+9}}$$

$$\theta = \cos^{-1}\left(\frac{-12}{\sqrt{9}\ \sqrt{50}}\right) = \boxed{124^\circ}$$ ⇒ Angle between the line & plane is 34°.

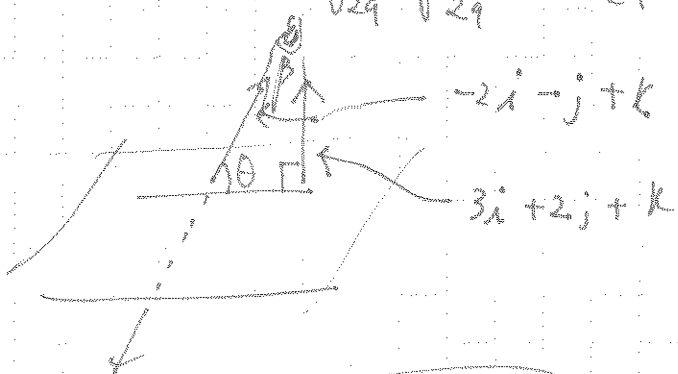
Acute Angle 56°

#5 $l_1 : \frac{x-3}{4} = \frac{y-1}{2} = \frac{z-2}{3}$ $\quad l_2 : r = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

$$\cos\theta = \frac{(4i + 2j + 3k) \cdot (3i + 4j - 2k)}{\sqrt{16+4+9}\ \sqrt{9+16+4}}$$

$$= \frac{12 + 8 - 6}{\sqrt{29}\ \sqrt{29}} = \frac{14}{29} \qquad \theta = \cos^{-1}\left(\frac{14}{29}\right) \approx \boxed{61.1^\circ}$$

#6.



$$\cos\beta = \frac{(-2i - j + k) \cdot (3i + 2j + k)}{\sqrt{4+1+1}\ \sqrt{9+4+1}}$$

$$\cos\beta = \frac{-6 - 2 + 1}{\sqrt{6}\ \sqrt{14}} = \cos\beta = \frac{-7}{\sqrt{6}\sqrt{14}}$$

$$\boxed{\theta = 90 - 40^\circ \approx 50^\circ}$$

$$\beta = \cos^{-1}\left(\frac{-7}{\sqrt{6}\ \sqrt{14}}\right) \approx 140^\circ$$

$$\beta = 40^\circ$$

#7. a. line 1 $\begin{cases} x = -7 + 15t \\ y = 2t \\ z = -4 + 3t \end{cases}$ $\quad \vec{D_1} = \boxed{15i + 2j + 3k}$

$\vec{D_2} = \boxed{5i + \frac{3}{2}j + k}$

line 2: $x = -3 + 5t \quad y = 2 + \frac{3}{2}t \quad z = 3 + t$

⇒

a) If two vectors are // $\Rightarrow$ Cross product is '0'. ③

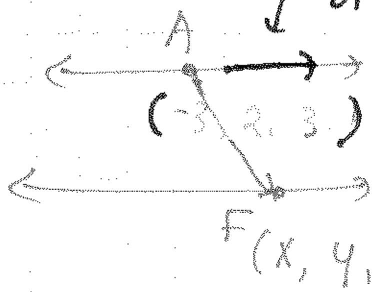
OR $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

$\Rightarrow \frac{15}{3} = \frac{2}{\frac{3}{2}} = \frac{3}{1}$

$\Rightarrow 3 = 3 = 3 \Rightarrow \therefore$ two lines are //.

$D_1 = 5i + \frac{3}{2}j + k$.

b)



$x = -7 + 15t$
$y = 2t$
$z = -4 + 3t$.

Shortest distance.

$D_1 \cdot \vec{AF} = 0$.

$$\vec{AF} = \begin{pmatrix} -7 + 15t + 3 \\ 2t - 2 \\ -4 + 3t - 3 \end{pmatrix} = \begin{pmatrix} 15t - 4 \\ 2t - 2 \\ 3t - 7 \end{pmatrix}$$

$\Rightarrow (5i + \frac{3}{2}j + k) \cdot ((15t - 4)i + (2t - 2)j + (3t - 7)k)$

$= ((75t - 20) + (3t - 3) + (3t - 7)) = 0$

$= 81t - 30 = 0 \quad t = \frac{30}{81} = \boxed{\frac{10}{27}}$

$$\vec{AF} = \begin{pmatrix} 15(\frac{10}{27}) - 4 \\ 2(\frac{10}{27}) \\ -4 + 3(\frac{10}{27}) \end{pmatrix} = \begin{pmatrix} \frac{14}{9} \\ \frac{20}{27} \\ \frac{-26}{9} \end{pmatrix}$$

$\Rightarrow \therefore$ the Shortest distance $\Rightarrow |\vec{AF}|$

$\Rightarrow \sqrt{(\frac{14}{9}) + (\frac{20}{27})^2 + (\frac{26}{9})^2} = \boxed{3.36 \text{ units}}$

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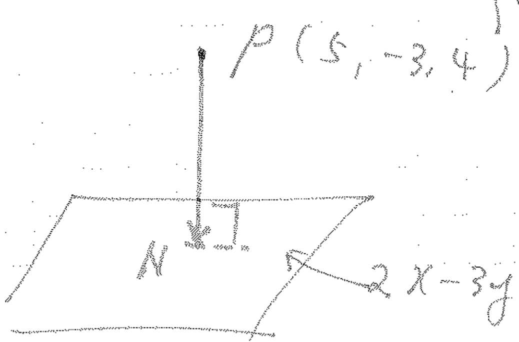
#9.



$$\vec{PN} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$2x - 3y + 4z = -1 \Leftarrow$
$x = 5 + 2t$
$y = -3 + (-3t)$
$z = 4 + 4t$

N

$x = 5 + 2(-\frac{36}{29}) = \frac{73}{29}$

$y = -3 + (-3)(-\frac{36}{29}) = \frac{21}{29}$

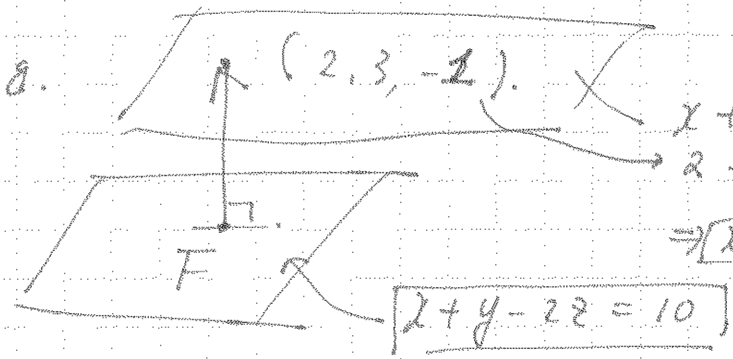
$z = 4 + (4)(-\frac{36}{29}) = \frac{-28}{...}$

$2(5 + 2t) - 3(-3 - 3t) + 4(4 + 4t) = -1$

$10 + 4t + 9 + 9t + 16 + 16t = -1$

$29t + 35 = -1 \quad 29t = -36 \quad \boxed{t = \frac{-36}{29}}$

#9.



$x + y - 2z = D$

$2 + 3 + 2 = 7$

$\Rightarrow \boxed{x + y - 2z = 7}$

Method 1: $D = \frac{10}{\sqrt{1+1+4}} - \frac{7}{\sqrt{1+1+4}} = \frac{3}{\sqrt{6}} = \boxed{\frac{3\sqrt{6}}{6}} = \boxed{\frac{\sqrt{6}}{2}} \approx \boxed{1.22}$

Method 2: $\vec{n} = i + j - 2k$.

line $\vec{r} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \begin{aligned} x &= 2 + t \\ y &= 3 + t \\ z &= -2 - 2t \end{aligned} \Rightarrow x + y - 2z = 10$

$(2+t) + (3+t) - 2(-2-2t) = 10$

$9 - 2t = 10 \quad 2t = -1 \quad t = -\frac{1}{2}.$

Intersection point F

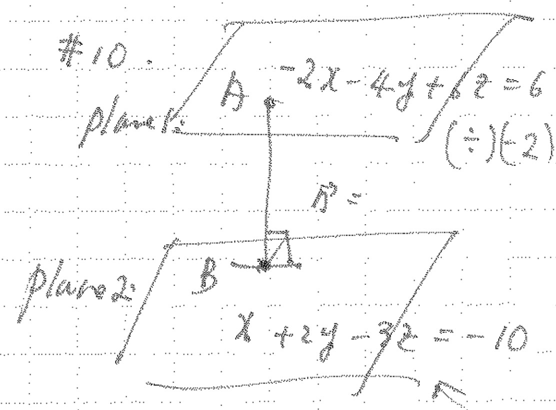
$x = 2 - \frac{1}{2} = \frac{3}{2}$

$y = 3 - \frac{1}{2} = \frac{5}{2}$

$z = -2 + 1 = -1$

$D = \sqrt{\left(2 - \frac{3}{2}\right)^2 + \left(3 - \frac{5}{2}\right)^2 + (-2+1)^2} = 1.22$

#10.



$\Rightarrow x + 2y - 3z = -3.$

Method 1: $D = \frac{-3 - (-10)}{\sqrt{1 + 4 + 9}} = \boxed{\frac{7}{\sqrt{14}}} \approx 1.87$

Method 2: $\vec{n} = i + 2j - 3k$.

point on plane 1: A: (0, 0, 1)

line $(\vec{r}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

$t + 2(2t) - 3(1 - 3t) = -10$

$5t - 3 + 9t = -10$

$14t = -7 \quad \boxed{t = -\frac{1}{2}.} \longrightarrow$

$x = t \quad x = -\frac{1}{2}$

$y = 2t \quad y = -1$

$z = 1 - 3t \quad z = 1 + \frac{3}{2} = \frac{5}{2}$

B: $\left(-\frac{1}{2}, -1, \frac{5}{2}\right)$

$|\vec{AB}| = \sqrt{\left(\frac{1}{2}\right)^2 + (1)^2 + \left(\frac{2}{2} - \frac{5}{2}\right)^2}$

$\approx \boxed{1.87 \text{ units}}$