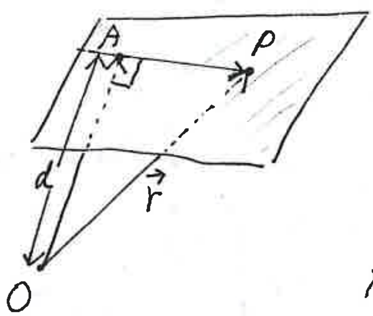
IB Math HL2 Distance line/line, line/plane, and plane/plane **Name** ________________

Distance from Origin to a plane using (a vector equation for a plane: $\vec{r}\cdot\hat{n}=d$)



Plane π

$P: (x, y, z)$

$A: (a, b, c)$

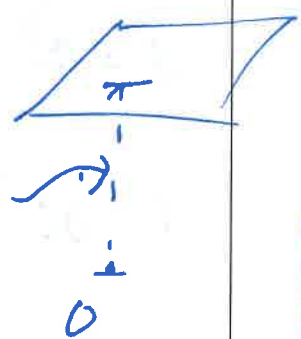
$\vec{r} = xi + yj + zk.$

- $\vec{AP} = \vec{AO} + \vec{r}$ (Given $\vec{AP} \perp \vec{OA}$) ①

$\vec{r}\cdot\hat{n}=d$

- $\hat{n}$ is an unit normal vector to the plane.
- $|OA| = d$ (Distance O to plane)
- $\overrightarrow{OA} = d\cdot\hat{n}$

$$\vec{r}\cdot\frac{\vec{n}}{|\vec{n}|} = \frac{\vec{a}\cdot\vec{n}}{|\vec{n}|} = d$$



- $\vec{OA} = |OA|\cdot\hat{n}$ ($|OA| = d$)
- $\vec{OA} = d\cdot\hat{n} \Leftrightarrow \vec{AO} = -d\cdot\hat{n}$ ②

$\Rightarrow \vec{AP}\cdot\vec{OA} = 0$ (Since $\vec{AP}\perp\vec{AO}$)

$\Rightarrow (\vec{AO}+\vec{r})\cdot\vec{OA} = 0$ (from ①)

$\Rightarrow \dfrac{(\vec{AO}+\vec{r})\cdot(-d\cdot\hat{n})}{d} = \dfrac{0}{d}$ (from ②)

$\Rightarrow (\vec{AO}+\vec{r})\cdot(+\hat{n}) = 0$

$\Rightarrow (-d\cdot\hat{n}+\vec{r})(\hat{n}) = 0$

$\Rightarrow -d\cdot\hat{n}\cdot\hat{n} + \vec{r}\cdot\hat{n} = 0$ $(\hat{n}\cdot\hat{n}=1)$

$\Rightarrow -d + \vec{r}\cdot\hat{n} = 0$

$\Rightarrow$ $\boxed{\vec{r}\cdot\hat{n} = d \text{ where } d \text{ is the distance from O to the plane.}}$

Distance:

EX) Given P1: $-3x+6y+7z=1$ and P2: $6x-12y-14z=25$.

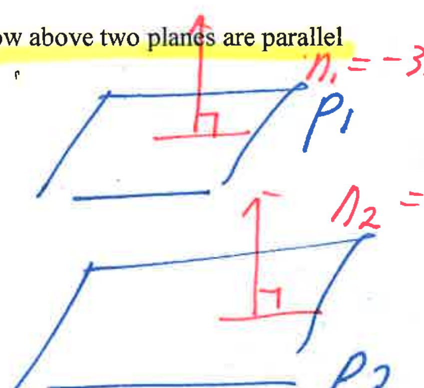
a) Show above two planes are parallel

$n_1 = -3i + 6j + 7k$

$n_2 = 6i - 12j - 14k$

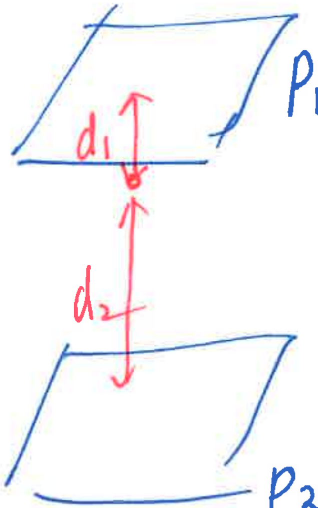
$\Rightarrow (-2)\vec{n}_1 = \vec{n}_2$

$\therefore P_1 \parallel P_2$.



b) Find the shortest distance between two planes.

b)



$$P_1 \Rightarrow \vec{r}\cdot\begin{pmatrix}-3\\6\\7\end{pmatrix} = 1 \qquad \div\sqrt{94}$$

$$\div\sqrt{9+36+49}$$

P_2

$$\Rightarrow \vec{r}\cdot\begin{pmatrix}6\\-12\\-14\end{pmatrix} = 25 \qquad \div(-2)$$

$$\div(-2)$$

$$\Rightarrow \vec{r}\begin{pmatrix}-3\\6\\7\end{pmatrix} = \frac{-25}{2} \qquad \div\sqrt{94}.$$

$$\div\sqrt{94}$$

$$P_1 : \vec{r}\left(\frac{-3i+6j+7k}{\sqrt{94}}\right) = \frac{1}{\sqrt{94}} = d_1$$

$$P_2 : \vec{r}\left(\frac{-3i+6j+7k}{\sqrt{94}}\right) = \frac{-25}{2\sqrt{94}} = d_2$$

$$d = |d_2 - d_1| = \left(\frac{1}{\sqrt{94}}\right) + \frac{25}{2\sqrt{94}}$$

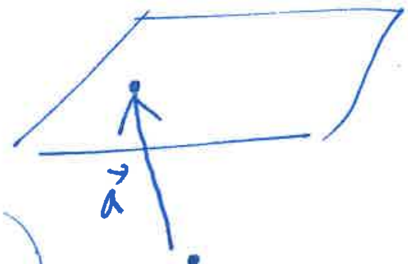
EX) Given A (1, -2, 1), B(1, 0, -2) and C (-2, 6, -6).

a) Find the equation of the plane determined by ABC. $\Rightarrow \vec{r} = \vec{a} + n\cdot\vec{u} + t\cdot\vec{v}$

$$\vec{u} = \vec{AB} = \begin{pmatrix} 1-1 \\ 0+2 \\ -2-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

$$\vec{r} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + n\begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 8 \\ -7 \end{pmatrix}$$

$$\vec{v} = \vec{AC} = \begin{pmatrix} -2-1 \\ 6+2 \\ -6-1 \end{pmatrix} = \begin{pmatrix} -3 \\ 8 \\ -7 \end{pmatrix}$$



b) Show the plane from part a is parallel to the line $x-3 = \frac{3-y}{2} = \frac{3z-3}{4}$.

$10i + 9j + 6k$

$10x + 9y + 6z = -2$

$\vec{n} = \vec{u} \times \vec{v}$

$$\Rightarrow \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ -3 & 8 & -7 \end{vmatrix} = i(-14+24) - j(-9) + k(6)$$

$= 10i + 9j + 6k$

$10x + 9y + 6z = D = -2$

$\cancel{10-12}$

$(1, 0, -2)$

$$x - 3 = \frac{y-3}{-2} = \frac{\cancel{3}z - 1}{\frac{4}{3}}$$

$\vec{D} = i - 2j + \frac{4}{3}k$

Show

$\vec{N} \perp \vec{d}$ by $\vec{N}\cdot\vec{d} = 0$

$\Rightarrow$ then plane // line.

$$\begin{pmatrix} 10 \\ 9 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ \frac{4}{3} \end{pmatrix} = 10 - 18 + 8 = 0$$

$\therefore$ plane // line

c) **Hence**, find the shortes distance from the line to the plane.

$$\vec{r}\begin{pmatrix} 10 \\ 9 \\ 6 \end{pmatrix} = -2$$

d_1

d_2

$$\vec{r}\begin{pmatrix} 10 \\ 9 \\ 6 \end{pmatrix} = 63$$

$$x - 3 = \frac{y-3}{-2} = \frac{z-1}{\frac{4}{3}}$$

$(3, 3, 1)$

$$\vec{a}\cdot\vec{n} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 9 \\ 6 \end{pmatrix}$$

$= 63$