Implicit Differentiation Practice

Problems 1 – 8: Find $\frac{dy}{dx}$.

1. $x^2 + 2y^2 = 1$
2. $y^7 = x^3(2-x)$
3. $7 = 2x^3 - 4y^5$
4. $\frac{x^2}{4} + y^6 = 1$
5. $\frac{1}{x} + \frac{1}{y} = 1$
6. $xy + 2x + 3x^2 = 4$
7. $3x^2y - xy + 5x^2 = y$
8. $xy^2 + 3x^3 = -5y$

9. Find the equation of the tangent to the curve $y^2 = \frac{y+3}{x-1}$ at $\left(\frac{5}{4}, 6\right)$.

10. Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ at $x = 9$.

Answers

1. $\frac{d}{dx}(x^2 + 2y^2) = \frac{d}{dx}(1)$

$2x + 4y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{2y}$

2. $\frac{d}{dx}(y^7) = \frac{d}{dx}(2x^3 - x^4)$

$7y^6\frac{dy}{dx} = 6x^2 - 4x^3$

$\frac{dy}{dx} = \frac{6x^2 - 4x^3}{7y^6}$

3. $\frac{d}{dx}(7) = \frac{d}{dx}(2x^3 - 4y^5)$

$0 = 6x^2 - 20y^4\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{3x^2}{10y^4}$

4. $\frac{d}{dx}\left(\frac{x^2}{4} + y^6\right) = \frac{d}{dx}(1)$

$\frac{1}{2}x + 6y^5\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{12y^5}$

5. $\frac{d}{dx}(x^{-1} + y^{-1}) = \frac{d}{dx}(1)$

$-x^{-2} - y^{-2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x^{-2}}{-y^{-2}} = -\frac{y^2}{x^2}$

6. $\frac{d}{dx}(xy + 2x + 3x^2) = \frac{d}{dx}(4)$

$1 \cdot y + x \cdot \frac{dy}{dx} + 2 + 6x = 0$

$x\frac{dy}{dx} = -y - 2 - 6x$

$\frac{dy}{dx} = \frac{-y - 2 - 6x}{x}$

7. $\frac{d}{dx}(3x^2y - xy + 5x^2) = \frac{d}{dx}(y)$

$6xy + 3x^2\frac{dy}{dx} - 1\cdot y - x\frac{dy}{dx} + 10x = \frac{dy}{dx}$

$(3x^2 - x - 1)\frac{dy}{dx} = -10x + y - 6xy$

$\frac{dy}{dx} = \frac{-10x + y - 6xy}{3x^2 - x - 1}$

8. $\frac{d}{dx}(xy^2 + 3x^3) = \frac{d}{dx}(-5y)$

$1\cdot y^2 + x\cdot 2y\frac{dy}{dx} + 9x^2 = -5\frac{dy}{dx}$

$(2xy + 5)\frac{dy}{dx} = -9x^2 - y^2$

$\frac{dy}{dx} = \frac{-9x^2 - y^2}{2xy + 5}$

9. $\frac{d}{dx}(y^2) = \frac{d}{dx}\left(\frac{y+3}{x-1}\right)$

$2y\frac{dy}{dx} = \frac{(x-1)\frac{dy}{dx} - (y+3)(1)}{(x-1)^2}$

At $\left(\frac{5}{4}, 6\right) \Rightarrow 2(6)\frac{dy}{dx} = \frac{\left(\frac{5}{4}-1\right)\frac{dy}{dx} - (6+3)}{\left(\frac{5}{4}-1\right)^2}$

$12\frac{dy}{dx} = \frac{\frac{1}{4}\frac{dy}{dx} - 9}{\frac{1}{16}}$

$\frac{3}{4}\frac{dy}{dx} = \frac{1}{4}\frac{dy}{dx} - 9$

$\frac{1}{2}\frac{dy}{dx} = 9$

$\frac{dy}{dx} = 18$

Tangent

$y = 18\left(x - \frac{5}{4}\right) + 6$

10. $\frac{d}{dx}\left(\sqrt{x} + \sqrt{y}\right) = \frac{d}{dx}(4)$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{1}{2}(9)^{-\frac{1}{2}} + \frac{1}{2}(1)^{-\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{1}{6} + \frac{1}{2}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{1}{3}$

$\sqrt{x} + \sqrt{y} = 4$

$x = 9 \Rightarrow \sqrt{9} + \sqrt{y} = 4$

$\sqrt{y} = 1$

$(9, 1)$ $y = 1$

Tangent

$y = -\frac{1}{3}(x - 9) + 1$