18J Higher Order Derivatives Name_____

<u>Higher Order Derivatives</u> Take the derivative again…and again…and again.

Given $f(x) = x^7 - 5x^6 + 43x^2 + 9$

The first derivative is $f'(x) = 7x^6 - 30x^5 +$ ____

The second derivative is $f''(x) =$ _____$-150x^4 + 86$

The third derivative is $f'''(x) = 210x^4 -$ _____

The fourth derivative is $f^{(4)}(x) =$ _____

(We could keep going, but I think you get the idea.)

<u>Notation</u>

If y is a function of x, then

$$y' = \frac{d}{dx}(y) = \frac{dy}{dx} \qquad y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \qquad y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

<u>Example 1</u>: Show that if $y = e^{3x^2+x}$, then $\frac{d^2y}{dx^2} = e^{3x^2+x}\left((6x+1)^2 + 6\right)$

<u>Example 2</u>: If $x^2 + y^2 = 4$ show that $\frac{d^2y}{dx^2} = -\frac{x^2+y^2}{y^3}$.

Step 1: Find $\frac{dy}{dx}$.

Step 2: Take the derivative of $\frac{dy}{dx}$. Leave your answer in terms of x and y.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\_\_\_\_\right)$$

Higher Order Derivatives Name Answer Key

Higher Order Derivatives Take the derivative again…and again…and again.

Given $f(x) = x^7 - 5x^6 + 43x^2 + 9$

The first derivative is $f'(x) = 7x^6 - 30x^5 + 86x$

The second derivative is $f''(x) = 42x^5 - 150x^4 + 86$

The third derivative is $f'''(x) = 210x^4 - 600x^3$

The fourth derivative is $f^{(4)}(x) = 840x^3 - 1800x^2$

(We could keep going, but I think you get the idea.)

Notation

If y is a function of x, then

$$y' = \frac{d}{dx}(y) = \frac{dy}{dx} \qquad y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \qquad y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

Example 1: Show that if $y = e^{3x^2+x}$, then $\frac{d^2y}{dx^2} = e^{3x^2+x}\left((6x+1)^2 + 6\right)$

Solution: $\frac{dy}{dx} = e^{3x^2+x}(6x+1)$

$$\frac{d^2y}{dx^2} = e^{3x^2+x}(6x+1)(6x+1) + e^{3x^2+x}(6)$$
$$= e^{3x^2+x}\left((6x+1)^2 + 6\right)$$

Example 2: If $x^2 + y^2 = 4$ show that $\frac{d^2y}{dx^2} = -\frac{x^2+y^2}{y^3}$.

Step 1: Find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}(4)$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-x}{y}$$

Step 2: Take the derivative of $\frac{dy}{dx}$. Leave your answer in terms of x and y.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-x}{y}\right) = \frac{y(-1) - (-x)\left(\frac{dy}{dx}\right)}{y^2}$$
$$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} \cdot \frac{y}{y}$$
$$= \frac{-y^2 - x^2}{y^3}$$
$$= -\frac{x^2 + y^2}{y^3}$$

Homework: 18J (1abd, 3, 5, 6, 7, 9, 11, 15bc)