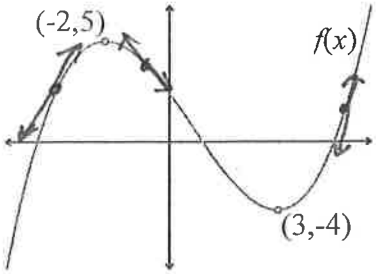
19B Increasing and Decreasing Functions

1. On what intervals is $f(x)$… a. Increasing? b. Decreasing?



a. $(-\infty, -2) \cup (3, \infty)$

$f'(x) > 0$

b. $(-2, 3)$

$f'(x) < 0$

2. On what intervals is $f(x) = 3x^4 + 4x^3 - 48x^2 - 144x + 36$ increasing? Decreasing?

$f'(x) > 0$ $f'(x) < 0$

$f'(x) = 12x^3 + 12x^2 - 96x - 144$.

$= 12(x^3 + x^2 - 8x - 12)$

possible zeros : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$.

$x = -2$ $f'(-2) = 12((-2)^3 + (-2)^2 - 8(-2) - 12) = 0$

$12(x+2)(x^2 - x - 6) = 12(x+2)(x+2)(x-3)$.

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -8 & -12 \\ & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

Sign diagram of $f'(x)$

$\ominus$ $\ominus$ $\oplus$

$x=-10$ -2 $x=0$ 3 $x=10$ x

$f'(x) = 12(x+2)^2(x-3)$

$\therefore$ Increasing : $(3, \infty)$

Decreasing : $(-\infty, -2) \cup (-2, 3)$

3. Find the greatest (absolute max) and least value (absolute min) of $y = x^3 - 6x^2 + 5$ on the interval $[2,5]$.

$y' = 3x^2 - 12x = 0$

$= 3x(x-4) = 0$

$\cancel{x = 0}$ $x = 4$

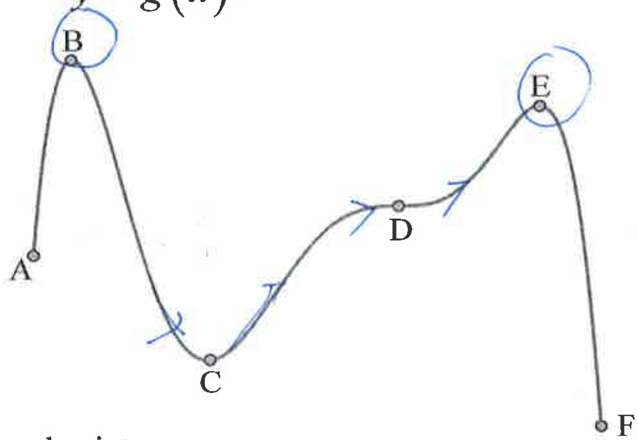
when | what is y value
$x = 4$ $y(4) = (4)^3 - 6(4)^2 + 5 = -27$ ⟸ Absolute min

$x = 2$ $y(2) = (2)^3 - 6(2)^2 + 5 = -11$ ⟸ Absolute Max

$x = 5$ $y(5) = (5)^3 - 6(5)^2 + 5 = -20$

19C Stationary Points

1. a. Consider the curve $y = g(x)$.



b. Label each lettered point as a
local maximum, local minimum, or stationary inflection.

$x = B$ and $x = E$ $\quad x = C \quad x = D$

c. Complete the sign diagram for the derivative of the function.

$g'(x)$: A $\oplus$ B $\ominus$ C $\oplus$ D $\oplus$ E $\ominus$ F

2. Complete the table

Stationary point where $f'(a) = 0$	Sign diagram of $f'(x)$ near $x = a$	Shape of curve near $x = a$
local maximum	$f'(x) > 0$ $x = a$ $f'(x) < 0$ | $\oplus \to \ominus$	
local minimum	$f'(x) < 0$ $x = a$ $f'(x) > 0$ | $\ominus \to \oplus$	
stationary inflection	$\oplus$ $x = a$. $\oplus$ | $\ominus$ $x = a$ $\ominus$	OR

3. Find and classify all stationary points of $f(x) = x^4 - 6x^2 + 8x + 1$.

$f'(x) = 0$

$f'(x) = 4x^3 - 12x + 8$

$= 4(x^3 - 3x + 2) = 4(x-1)(x^2 + x - 2)$

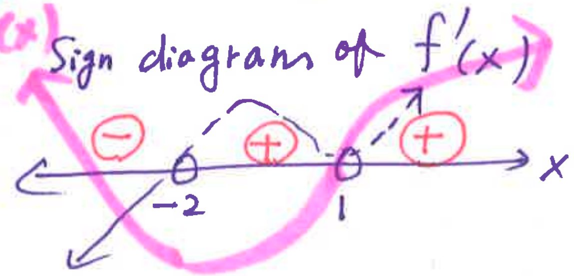
possible : $\pm 1, \pm 2$. $\quad = 4(x-1)(x-1)(x+2)$

$f'(1) = (4)(1 - 3 + 2) = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

stationary points: $x = 1 \quad x = -2$

$f(x)$ Sign diagrams of $f'(x)$

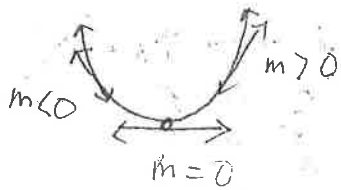


- $x = -2$ $\searrow \nearrow$ Local min.
- $x = 1$ $\nearrow \nearrow$ Stationary Inflection.
- None. Local Max

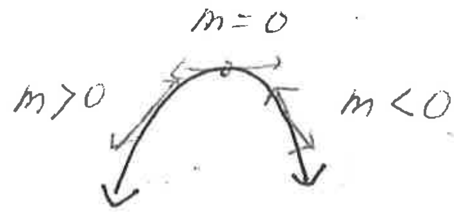
19D.1 Concavity　　　　　　　　　　Name________________________________

1. Sketch functions as indicated. Fill in the blanks with increasing, decreasing, positive, or negative.

a. Sketch a function $f(x)$ that is concave up.



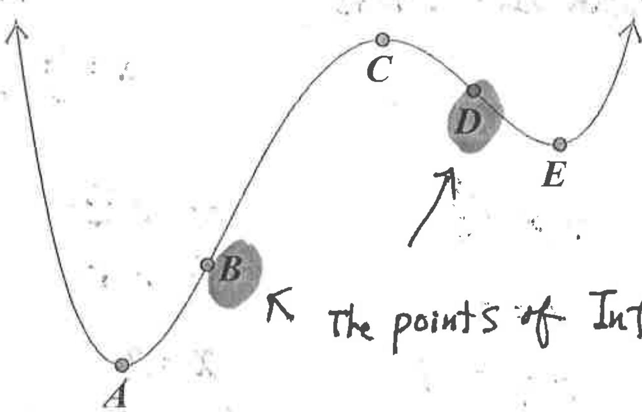
b. Sketch a function $g(x)$ that is concave down.



$f'(x)$ is $\ominus \rightarrow \oplus$　　$f''(x)$ is $\oplus$　$\frac{d^2f}{dx^2}$

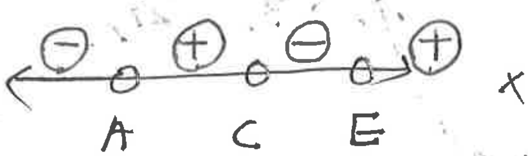
$g'(x)$ is $\oplus \rightarrow \ominus$　　$g''(x)$ is $\ominus$　$\frac{d^2g}{dx^2}$

2. Consider the function $h(x)$.



The points of Inflection

Make a sign diagram for $h'(x)$.



Increasing : $(A, C) \cup (E, \infty)$

Decreasing : $(-\infty, A) \cup (C, E)$

Make a sign diagram for $h''(x)$.

Concave up : $(-\infty, B) \cup (D, \infty)$

Concave down : (B, D)

19D.1 Concavity Name________________________________

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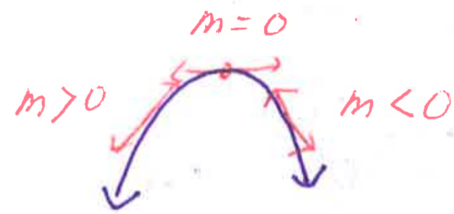
b. Sketch a function $g(x)$ that is concave down.



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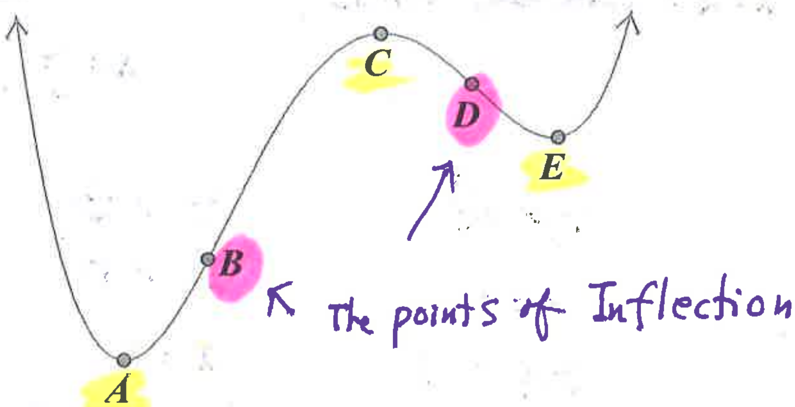
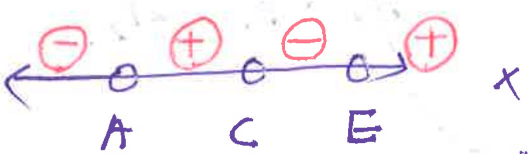


Make a sign diagram for $h'(x)$.



Increasing : $(A, C) \cup (E, \infty)$

Decreasing : $(-\infty, A) \cup (C, E)$

Make a sign diagram for $h''(x)$.

Concave up : $(-\infty, B) \cup (D, \infty)$

Concave down : (B, D)