1st period

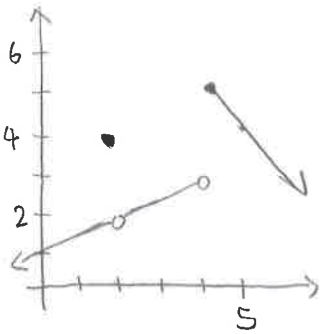
Final Review Notes

2016 ~ 2017

CHAPTER 17

limit: $\lim_{x \to a} f(x) = A$ as x approaches a, $f(x)$ converges on A

Ex1: Evaluate each expression for the given graph $f(x)$.



a. $\lim_{x \to 2} f(x) = 2$

b. $\lim_{x \to 4} f(x) = \text{DNE}$

c. $\lim_{x \to 4^-} f(x) = 3$

d. $\lim_{x \to 2^-} f(x) = 2$

e. $f(4) = 5$

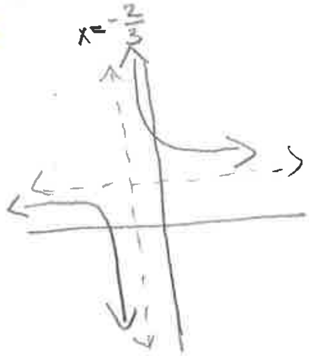
f. $f(2) = 4$

Limits to Infinity:

Ex2 $\lim_{x \to -\infty} \frac{1+2x}{3x+2}$

HA $= \frac{2}{3}$

VA $= -\frac{2}{3}$



as x approaches $-\infty$, y approaches $\frac{2}{3}$

$$\lim_{x \to -\infty} \frac{1+2x}{3x+2} = \frac{2}{3}$$

Solving Limits:

Ex3 Evaluate: $\lim_{x \to 3} \frac{x^2-9}{x-3}$

$$= \lim_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} (x+3)$$

$$= 6$$

Trig Limits:

Ex4 (use trig identities)

note: $\frac{\sin x}{x} = 1$

$$\lim_{x \to 0} \frac{1-\cos 2x}{5x^2}$$

$$\lim_{x \to 0} \frac{\sin 2x}{5x^2}$$

$$\lim_{x \to 0} \frac{\sin x}{5x} \quad \frac{\sin x}{x}$$

$$\lim_{x \to 0} \frac{1}{5} \frac{\sin x}{x}$$

$$= \frac{1}{5}$$

Definition of Derivative:

$f'(a)$ is the slope of the tangent to $y = f(x)$ at the point where $x = a$

★ First principle

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Def. of Derivative Cont.

Ex 5

Find the slope of the tangent line to the graph of $f(x) = -2x^2 + 5x$ at $x = 3$ using the definition of the derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\lim_{h \to 0} \frac{[-2(3+h)^2 + 5(3+h)] - (-2(3)^2 + 5(3))}{h}$$

$$\lim_{h \to 0} \frac{[-2(h^2 + 6h + 9) + 15 + 5h] - (-18 + 15)}{h}$$

$$\lim_{h \to 0} \frac{-2h^2 - 12h - 18 + 15 + 5h + 3}{h}$$

$$\lim_{h \to 0} \frac{-12h - 2h^2 + 5h}{h}$$

$$\lim_{h \to 0} \frac{h(-12 - 2h + 5)}{h}$$

$$\lim_{h \to 0} -2h - 7$$

$$\lim_{h \to 0} -7 = \boxed{-7}$$

GROUP 2

CHAPTER 18: DERIVATIVES

RULES

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = \frac{dy}{dx}$$

PRODUCT RULE:

If $f(x) = u(x)\cdot v(x)$, then

$f'(x) = u'(x)\cdot v(x) + u(x)\cdot v'(x)$

POWER RULE:

If $f(x) = x^n$, then

$f'(x) = (n)x^{n-1}$

CHAIN RULE:

If $y = (f(x))^n$, then

$\frac{dy}{dx} = n(f(x))^{n-1}$

QUOTIENT RULE:

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{vu' - uv'}{v^2}$$

CONTINUITY + DIFFERENTIABILITY

Function f is differentiable if and only if it is differentiable at every value of x in its domain.

$f: D \rightarrow R$ is differentiable at $x=a$, if

1) f is continuous at $x=a$
2) f is differentiable at $x=a$ if

$$f'(a) = \lim_{h\to 0^+} \frac{f(a+h)-f(a)}{h} \text{ (right hand derivative) and}$$

$$f'(a) = \lim_{h\to 0^-} \frac{f(a+h)-f(a)}{h} \text{ (left hand derivative)}$$

both exists and are equal.

Continuity: If f is ~~differentiabilit~~ differentiable at $x=c$, then f is continuous at $x=c$.

EXAMPLES:

1) Differentiate:

$$f(x) = \frac{12}{\sqrt[4]{x^3}} - \frac{1}{2}x^{10} = 12x^{-\frac{3}{4}} - \frac{1}{2}x^{10}$$

It can be helpful to rewrite the function to make it easier to differentiate.

$$f'(x) = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-36}{4}x^{-\frac{7}{4}} - \frac{10}{2}x^{9}$$

Power Rule

$$\boxed{\frac{dy}{dx} = \underbrace{\frac{-9}{\sqrt[4]{x^7}}}_{\text{simplify}} - 5x^{9}}$$

2) The function f is defined by:

$$f(x) = \begin{cases} 2x-1 & x \leq 2 \\ ax^2+bx-5 & 2 < x < 3 \end{cases}$$

a) If f and f' are continuous, find a and b.

f(x)

$$2x-1 = ax^2+bx-5$$
$$2(2)-1 = a(2)^2+2b-5$$
$$4 = -4a$$
$$a=-1$$

f'(x)

$$2 = 2ax+b$$
$$2-4a=b$$
$$2-4(-1)=b$$
$$b=6$$

Curve Analysis

What's it used for?: Finding the maximum, minimum, and trend of a certain graph/equation.

Increasing/Decreasing Functions:

This will help determine function's domains. The rules are as follows:

$f(x)$ is increasing on S^* $f'(x) \geq 0$ for all x in S

$f(x)$ is strictly increasing on $S \Leftrightarrow f'(x) > 0$ for all x in S.

$f(x)$ is decreasing on $S \Leftrightarrow f'(x) < 0$ for all x in S

$f(x)$ is strictly decreasing on $S \Leftrightarrow f'(x) < 0$ for all x in S

* Given S is an interval.

Sign Diagrams for the derivatives are used for helping you determine increasing/decreasing functions.

Ex: $f(x) = x^3 - 3x + 4$

$f'(x) = 3x^2 - 3$

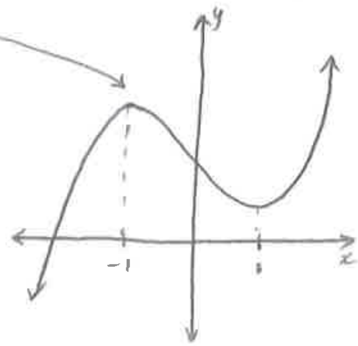
$= 3(x+1)(x-1)$

sign diagram:

$+ \quad - \quad +$

$-1 \quad 1$

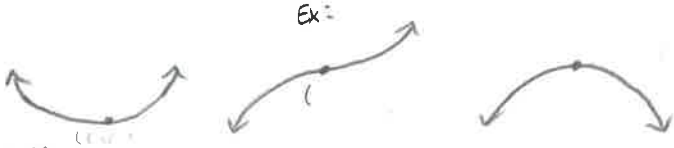
(find the increase/decrease by plugging in numbers that 'fit' within each interval).



Stationary Points:

A stationary point is a point where $f'(x) = 0$. These points signify a turn in the graph.

Ex:



There are multiple different stationary points:

A: a global minimum. It is the smallest y value in the entire domain.

B: a local maximum. It is a turning point where $f'(x) = 0$ and takes the shape: ∩

C: a local minimum. It is a turning point where $f'(x) = 0$ and takes the shape ∪

D: a global maximum. It has the largest y value in the entire domain.

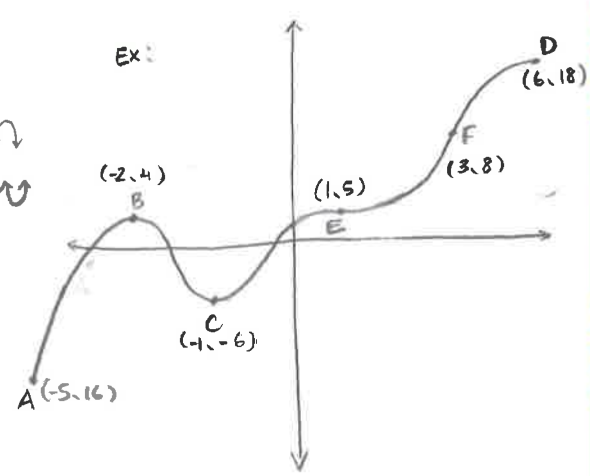
In some situations, the global max./min. is the same as the local min./max.

Ex: $f(x) = x^2$, where $x = 0$ is both the global and local minimum.

E: a stationary point of inflexion.

F: a point of inflexion.

Ex:

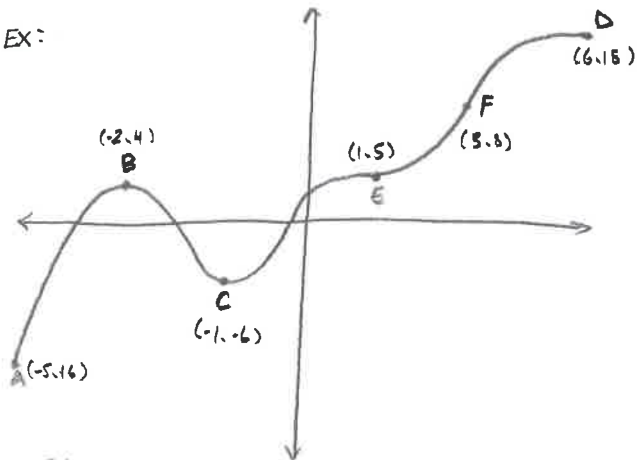


Sign Diagrams (In Relation to Stationary / Turning Points):

Sign Diagrams can be used to differentiate and characterise stationary / turning points.

Stationary Point where $f'(a)=0$	Sign Diagram of $f'(x)$ near $x=a$	Shape of Curve near $x=a$
local max.	$+ \mid -$; a	$x=a$
local min.	$- \mid +$; a	$x=a$
Inflexion.	$- \mid -$; a OR $+ \mid +$; a	$x=a$ OR $x=a$

EX:



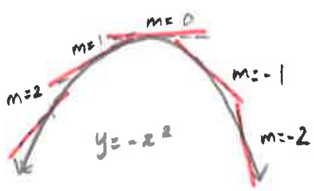
Point B: $+ \mid -$; -2

Point E: $+ \mid +$; 1

Point of Inflexions also determine concavity.

This means that as x increases, the gradient of the tangent decreases...

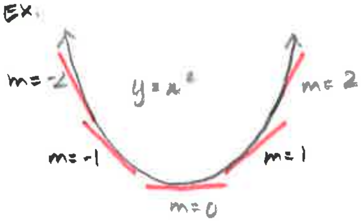
Ex.



And results in a concave downwards.

And as x increases and the gradient of the tangent increases...

Ex.



It results in a concave upwards.

Examples are missing.

Group 3 Period 1: Natalia, Andre, Angela, and Naomi

Trig Derivatives

$\sin x \to \cos x$
$\cos x \to -\sin x$
$\tan x \to \sec^2 x$
$\sec x \to \sec x \tan x$
$\csc x \to -\csc x \cot x$
$\cot x \to \csc x$

example 1

$y = \cos(2x)$

$dy/dx = 2\sin(2x)$

example 2

$y = x\sin x$

$dy/dx = (1)\sin x + x(\cos x)$

$dy/dx = \sin x + x\cos x$

example 3

$y = \tan^3(5x)$

$y = (\tan(5x))^3$

$dy/dx = 3(\tan(5x))^2 \cdot \sec^2(5x)(5)$

$dy/dx = 3\tan^2(5x) \cdot 5\sec^2(5x)$

Inverse Trig Derivatives

$\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$

$\arccos x = \dfrac{-1}{\sqrt{1-x^2}}$

$\arctan x = \dfrac{1}{1+x^2}$

$\operatorname{arcsec} x = \dfrac{1}{|x|\sqrt{x^2-1}}$

$\operatorname{arccsc} x = \dfrac{-1}{|x|\sqrt{x^2-1}}$

example 1

$y = \sin^{-1}(4x^3)$

$dy/dx = 12x^2\left(\dfrac{1}{\sqrt{1-(4x^3)^2}}\right)$

$dy/dx\ \dfrac{12x^2}{\sqrt{1-16x^6}}$

example 2

$y = 5^x \csc^{-1} x$

$dy/dx = 5^x(\ln 5)\csc^{-1}x + \dfrac{-1}{|x|\sqrt{x^2-1}}(5^x)$

$dy/dx = 5^x\left(\ln 5 \csc^{-1} x - \dfrac{1}{|x|\sqrt{x^2-1}}\right)$

Log Derivatives

$\ln x = \dfrac{1}{x}$

example 1

$y = \ln(1-3x)$

$dy/dx = \dfrac{-3}{1-3x}$

example 2

$y = \ln\left(\dfrac{x^2}{(x+2)(x-3)}\right)$

$dy/dx = \ln x^2 - \ln(x+2) - \ln(x-3)$

$dy/dx = 2\ln x - \ln(x+2) - \ln(x-3)$

$dy/dx = \dfrac{2}{x} - \dfrac{1}{x-2} - \dfrac{1}{x-3}$

$\log_a x = \dfrac{1}{x\ln a}$

example 1

$y = \log_2 5x$

$dy/dx = \dfrac{5}{5x\ln 2}$

$dy/dx = \dfrac{1}{x\ln 2}$

$e^x = e^x$

example 1

$y = 2e^x + e^{-3x}$

$y = 2e^x - 3e^{-3x}$

Equation of Tangent and Normal lines

tangent $\to$ $y - f(c) = f'(c)(x-c)$

Find equation of the tangent line to the graph of $f(x) = x^3(2-3x)^2$ at $x=1$

$f(x) = x^3(2-3x)^2$

$f(1) = 1^3(2-3(1))^2$

$f(1) = 1(-1)^2$

$f(1) = 1$

$f'(x) = 3x^2(2-3x)^2 + 2(2-3x)(-3)(x^3)$

$f'(x) = 3x^2(2-3x)^2 - 6x^3(2-3x)$

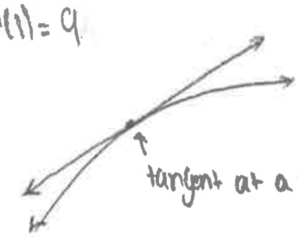
$f'(1) = 3(1)^2(2-3(1))^2 - 6(1)^3(2-3(1))$

$f'(1) = 3(-1)^2 - 6(1)(-1)$

$f'(1) = 3+6$

$f'(1) = 9$

$y - 1 = 9(x-1)$



normal $\to$ $y - f(c) = \dfrac{-1}{f'(c)}(x-c)$

Find the equation(s) of the normal line to the graph of $f(x) = \sqrt{5x}$ at $x=5$

$f(x) = \sqrt{5x}$

$f(5) = \sqrt{5 \cdot 5}$

$f(5) = 5$

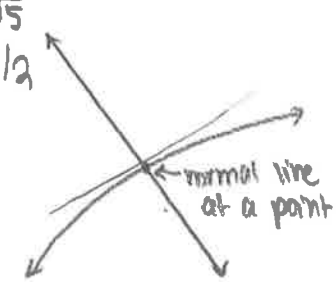
$f'(x) = 1/2(x)^{-1/2}(\sqrt{5})$

$f'(x) = \dfrac{\sqrt{5}}{2\sqrt{x}}$

$f'(5) = \dfrac{\sqrt{5}}{2\sqrt{5}}$

$f'(5) = 1/2$

$y - 5 = -2(x-5)$



Group 6: Anjali Agarwal, Pankhuri Singhal, Adam Dong, Mike Zhang

Optimization:

① Draw a diagram

② Write an equation that represents the situation

③ Find the derivative of that equation & set it equal to 0

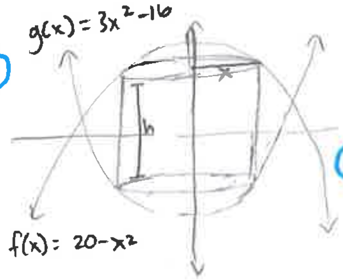
→ Why? Because optimization = Finding max & min points & values

④ Solve for x when derivative = 0

→ If the question is asking for y value, plug in x values into original function.

Example 1:

①



What is the maximum volume of the cylinder?

$V=\frac{1}{3}\pi r^2 h$ [cm^3]

$\rightarrow V=\frac{1}{3}\pi x^2(20-x^2-(3x^2-16))$

$\rightarrow V=\frac{\pi}{3}x^2(20-4x^2+16) = \frac{\pi}{3}x^2(36-4x^2)=V$ ②

$\therefore V=12\pi x^2-\frac{4\pi}{3}x^4$

③ $\frac{dV}{dx}=24\pi x-\frac{16\pi}{3}(x^3)=0 \rightarrow$ ④ $x(24\pi-\frac{16\pi x^2}{3})=0$

$=-24\cancel{\pi}=\frac{-16\cancel{\pi}x^2}{3} \rightarrow +24\left(\frac{3}{16}\right)=x^2 \rightarrow x^2=\frac{9}{2}$

$x=\pm\sqrt{\frac{9}{2}}$

$V=\frac{1}{3}\pi\left(\frac{9}{2}\right)\left(36-4\left(\frac{9}{2}\right)\right) \rightarrow V=\frac{3\pi}{2}(36-18)=\frac{3\pi}{2}\times 18$

$=27\pi\ cm^3$

Related Rate:

① Draw a diagram

② seperate information into

A. → A rate at which something is changing

Height of cone is increasing at rate of 3 cm/s $\frac{dh}{dt}=3$ cm/s

→ A constant value or relationship

B. Radius is always half the height in this cone $r=\frac{h}{2}$

→ A value at a specific time that you will substitute in

C. What is the rate at which volume is changing when $h=5$ cm

③ Write an equation to represent the situation

* → DO NOT substitute C. values in, or A. values in

→ only substitute B. values in

④ Multiply all sides by $\frac{d}{dt}$

→ $V=\frac{1}{3}\pi r^2 h$

$\frac{dV}{dt}=\frac{\pi}{3}\cdot 2r\cdot\frac{dr}{dt}\cdot\frac{dh}{dt}$

⑤ Now substitute values in!

→ Solve for rate or value

Example 2: Solve for $\frac{dV}{dt}$ when $h=4$ cm

①

$2r=h$ $\quad \frac{dh}{dt}=80$ cm/min ②

$V=\frac{1}{3}\pi r^2 h$ ③

$V=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2\cdot h$

$V=\frac{1}{3}\pi\cdot\frac{h^2}{4}\cdot h$

$V=\frac{\pi}{12}h^3$

④ $\frac{dV}{dt}=\frac{\pi}{12}\cdot 3h^2\cdot\frac{dh}{dt}$

$\frac{dV}{dt}=\frac{\pi}{12}\cdot 3h^2\cdot 80$

⑤ $\frac{dV}{dt}=\frac{\pi}{4}\cdot 80\cdot h^2$

$\frac{dV}{dt}=20\pi\cdot h^2$

$\frac{dV}{dt}=20\pi\cdot(4)^2=\boxed{320\pi}$