Preston Foust
Dhruv Gupta
Cory Y Yoshimura
Samprikta Basu

1. Find values of a and b so that $D(1,3,a)$, $E(2,4,6)$ and $F(6,b,12)$ are collinear.

SOLUTION: $\overrightarrow{DE} = \cancel{k}\ t\ \overrightarrow{EF}$ +1 initial equation

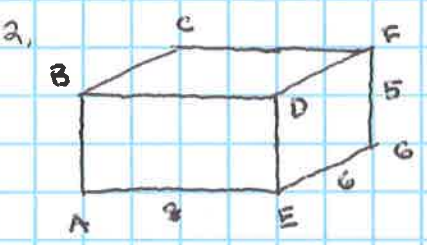
$\overrightarrow{DE} = \begin{pmatrix} 2-1 \\ 4-3 \\ 6-a \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 6-a \end{pmatrix}$

$1 = 6t$
$1 = t(b-4)$
$6 - a = 10t$

+1 for algebraic solving set up

$\boxed{\begin{array}{l} t = 1 \\ b = 4 \\ a = -4 \end{array}}$ +1 for answers

2.



a) Find vectors $\vec{CE}$ and $\vec{AF}$

SOLUTIONS: $\vec{CE} = 6i + 8j - 5k$
$\vec{AF} = -6i + 8j + 5k$

+1 for finding the vectors

b) Express the angle between the two diagonals in radians.

SOLUTION: $\vec{CE} = \sqrt{6^2 + 8^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$
$\vec{AF} = \sqrt{6^2 + 8^2 + 5^2} = \sqrt{125} = 5\sqrt{5}$

+1 for algebraic solving

$CE \cdot AF = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 8 \\ 5 \end{pmatrix} = -36 + 64 - 25 = \boxed{3}$

$\cos\theta = \dfrac{\vec{CE} \cdot \vec{AF}}{|\vec{CE}|\,|\vec{AF}|}$

+1 for correct setup

$\theta = \arccos\left(\dfrac{3}{10\sqrt{5}}\right) = 1.43623$
$\boxed{1.43}$

+1 for answer

3. Given 3 points: $A(5,6,4)$, $B(2,0,3)$, $C(10,6,1)$
Find $\vec{AB} \cdot \vec{AC}$

SOLUTION: $\vec{AB} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ -1 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 10 \\ 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix}$

+1 for finding vectors

$\Rightarrow \vec{AB} \cdot \vec{AC} = \begin{pmatrix} -3 \\ -6 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ -3 \end{pmatrix} = -15 + 0 + 3$
$= \boxed{-12}$

+1 for dot product

Math Quiz Questions

Arnav [illegible]., Zan C.,
Ashley C.

1). Find x and y so that $\vec{a}$ and $\vec{b}$ are parallel.

$$\vec{a} = \begin{pmatrix} 4 \\ 5 \\ x \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 2 \\ y \\ 6 \end{pmatrix}$$

$\vec{a} = t\vec{b}$

$4 = t(2)$ $5 = t(y)$ $x = t(6)$

$t = 2 \longrightarrow$ $5 = 2(y)$ $x = 2(6)$

$\boxed{y = \frac{5}{2}}$ $\boxed{x = 12}$

Rubric: /4

+1 = identifying proper equation

+1 = finding "t"

+1 = finding "y"

+1 = finding "x"

2). Given $\vec{a} = 6i - 9j + wk$ and $\vec{b} = 9i + 7j - 4k$, $\vec{a} + 4\vec{b} = \begin{pmatrix} 42 \\ 37 \\ 2 \end{pmatrix}$, find w.

$$\vec{a} + 4\vec{b} = \begin{pmatrix} 6 \\ 9 \\ w \end{pmatrix} + 4\begin{pmatrix} 9 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ w \end{pmatrix} + \begin{pmatrix} 36 \\ 28 \\ -16 \end{pmatrix} = \begin{pmatrix} 42 \\ 37 \\ w-16 \end{pmatrix}$$

$w - 16 = 2 \rightarrow \boxed{w = 18}$

Rubric: /2

+1 = set-up

+1 = finding "w"

3). Find the angle between:

$$\vec{a} = \begin{pmatrix} 8 \\ 4 \\ -6 \end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$$

$\vec{a} \cdot \vec{b} = -16 + 4 - 30 = -18$

$|\vec{a}| = \sqrt{64 + 16 + 36} = \sqrt{116}$

$|\vec{b}| = \sqrt{4 + 1 + 25} = \sqrt{30}$

$$\cos\theta = \frac{-18}{\sqrt{116}\sqrt{30}}$$

$$\theta = \arccos\left(\frac{-18}{\sqrt{116}\sqrt{30}}\right)$$

$\boxed{\theta = 1.88 \text{ radians}}$

Rubric:

+1 = proper equation

+1 = dot product

+1 = solving $|\vec{a}|$

+1 = solving $|\vec{b}|$

+1 = correct answer

Ricky Z, Anirudh K, Sophie Z

1. Find k given that $\begin{pmatrix} k \\ \frac{1}{\sqrt{2}} \\ -k \end{pmatrix}$ is a unit vector

$$\sqrt{k^2+\left(\tfrac{1}{\sqrt{2}}\right)^2+(-k)^2} = 1$$

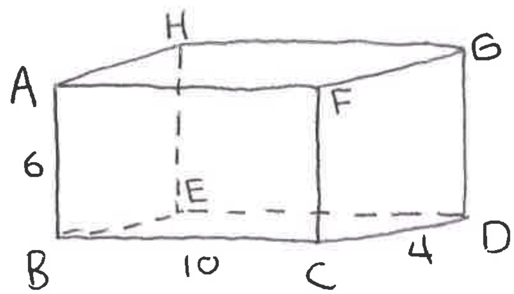
$$\left(\sqrt{k^2+\tfrac{1}{2}+k^2} = 1\right)^2$$

$$k^2+\tfrac{1}{2}+k^2=1$$

$$2k^2=\tfrac{1}{2}$$

$$\boxed{k=\pm\tfrac{1}{2}}$$

Rubric:
+1 for setting up equation = 1
+1 for solving for k

2. 

Find vector in direction of $\overrightarrow{AD}$ with magnitude of 5.

$$\overrightarrow{AD} = 4i-10j+6k$$

$$\frac{4i-10j+6k}{\sqrt{(4)^2+(10)^2+(6)^2}} = \boxed{\frac{4i-10j+6k}{\sqrt{152}}}$$

Rubric for #2
+1 = find $\overrightarrow{AD}$
+1 = find magnitude of $|AD|$
+0.5 = find $\frac{\overrightarrow{AB}}{|AB|}$ correctly

3. Find t if the angle between $\begin{pmatrix} 4t \\ t-1 \\ -3t \end{pmatrix}$ and $\begin{pmatrix} t+3 \\ t \\ t \end{pmatrix}$ is 90°

$$\cos(90) = \frac{p \cdot q}{|p||q|}$$

$$\cos(90) = \frac{4t^2+12t+t^2-t-3t^2}{\left(\sqrt{26t^2-2t+1}\right)\left(\sqrt{3t^2+6t+9}\right)}$$

$$\cos(90) = 2t^2+11t$$

$$0 = 2t^2+11t$$

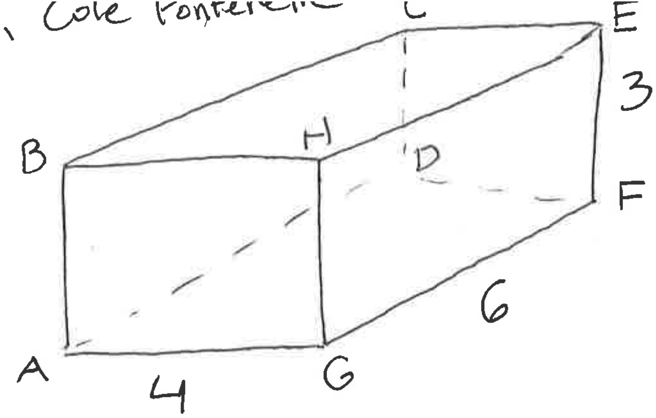
$$0 = t(2t+11)$$

$$\boxed{t=0} \qquad \boxed{t=-\tfrac{11}{2}}$$

Rubric for #3
+2 = find both values of t
+1 = set up equation correctly
+

Lauren Heiberg, Haydn Fitzgerald, Cole Fonterelle

1. Find the angle between $\overrightarrow{AC}$ and $\overrightarrow{AF}$



$\overrightarrow{AC}\begin{pmatrix}-6\\0\\3\end{pmatrix}$ $\overrightarrow{AF}\begin{pmatrix}-6\\4\\0\end{pmatrix}$

$\overrightarrow{AF}\cdot\overrightarrow{AC} = (-6)(-6)+(4)(0)+(0)(3) = 36$

$|\overrightarrow{AF}||\overrightarrow{AC}| = \sqrt{(-6)^2+4^2}\sqrt{(-6)^2+3^2} = \sqrt{45}\sqrt{52}$
$= 3\sqrt{5}\,2\sqrt{13}$

$\dfrac{\overrightarrow{AF}\cdot\overrightarrow{AC}}{|\overrightarrow{AF}||\overrightarrow{AC}|} = \cos\theta$

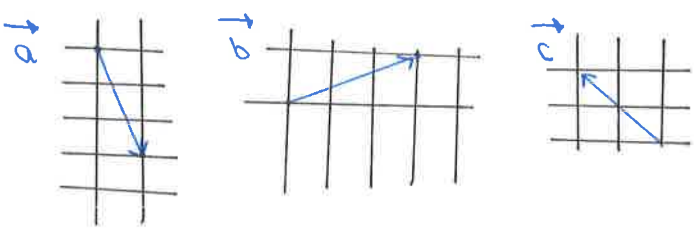
$\dfrac{36}{3\sqrt{5}\,2\sqrt{13}} = \cos\theta$ $\boxed{\theta \approx 41.9°}$

2. If ~~$\vec{a}\begin{pmatrix}4\\-3\\5\end{pmatrix}$ and $\vec{b}\begin{pmatrix}-2\\t\\6\end{pmatrix}$~~ $\vec{a}\begin{pmatrix}3\\2\\4\end{pmatrix}$ and $\vec{b}\begin{pmatrix}-2\\t\\6\end{pmatrix}$ are perpendicular, find t

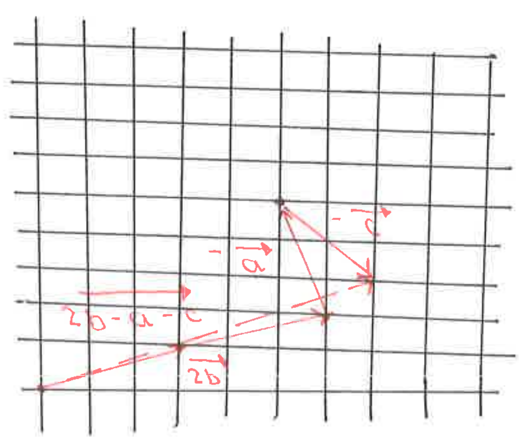
$\begin{pmatrix}3\\2\\4\end{pmatrix}\cdot\begin{pmatrix}-2\\t\\6\end{pmatrix} = \begin{pmatrix}-6\\2t\\24\end{pmatrix}$

$-6+2t+24=0$
$2t+18=0$
$2t=-18$
$\boxed{t=-9}$

3. given...



construct 2b-a-c geometrically



[illegible] sarode   Yifan Xu
awaniy chakravarty   arkhip moroz

1. suppose $u = 2i + j$, $v = 3j$, and θ is an acute angle between u and v. what is the exact value of $\sin\theta$?

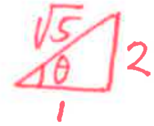
$u = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|u||v|}$

$v = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$u \cdot v = (2 * 0) + (1 * 3) = 3$

$|u| = \sqrt{5}$

$|v| = 3$



$\cos\theta = \dfrac{3}{\sqrt{5} * 3} = \dfrac{1}{\sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$

$\therefore \sin\theta = \sin\left(\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)\right) = \boxed{\cancel{0.89}}$ $\dfrac{2}{\sqrt{5}}$

2. Shim and Mish are running in straight lines. They start at the point (5, 4) at the same time. After a minute, Shim is at (12, 2) and Mish is at (7, 10).

a. how far is each person from their starting point?

$S = \begin{pmatrix} 12-5 \\ 2-4 \end{pmatrix} = \boxed{\begin{pmatrix} 7 \\ -2 \end{pmatrix}}$ $m = \begin{pmatrix} 7-5 \\ 10-4 \end{pmatrix} = \boxed{\begin{pmatrix} 2 \\ 6 \end{pmatrix}}$

b. what is the angle between the path of these two?

$S \cdot m = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 14 - 12 = 2$ $\cos\theta = \dfrac{s \cdot m}{|s| \cdot |m|} = \dfrac{2}{\sqrt{2^2+7^2}\sqrt{6^2+2^2}} = \dfrac{2}{\sqrt{53}\sqrt{40}} = \dfrac{1}{\sqrt{530}}$

$\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{530}}\right) = \boxed{87.5^\circ}$

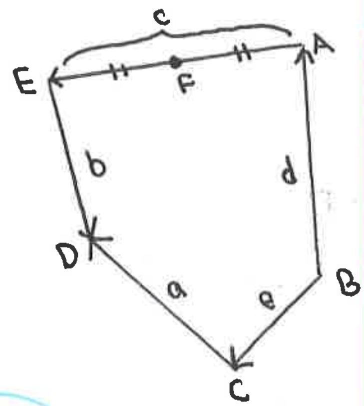
c. if shim keeps running so she is 20 units from the starting point, where is she on the coordinate plane?

$20 \cdot \dfrac{S}{|S|} = 20 \cdot \dfrac{-2i + 7j}{\sqrt{53}} = -\dfrac{40}{\sqrt{53}} i + \dfrac{140}{\sqrt{53}} j$

$\begin{pmatrix} 5 \\ 4 \end{pmatrix} + \begin{pmatrix} -\frac{40}{\sqrt{53}} \\ \frac{140}{\sqrt{53}} \end{pmatrix} = \begin{pmatrix} 5 - \frac{40}{\sqrt{53}} \\ 4 - \frac{140}{\sqrt{53}} \end{pmatrix}$

$$\boxed{\left(5 - \frac{40}{\sqrt{53}},\ 4 - \frac{140}{\sqrt{53}}\right)}$$

3. assume point F is the midpoint for $\overrightarrow{AE}$



a. write $\overrightarrow{BD}$ in terms of a and e.

$\overrightarrow{BD} = e + a$

b. write $\overrightarrow{CF}$ ...

i. in terms of c, d, and e.

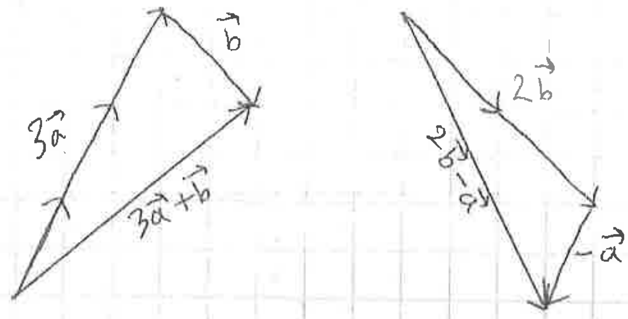
$\overrightarrow{CF} = -e + d + \frac{1}{2}c$

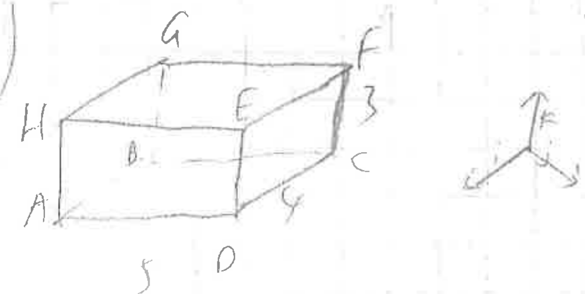
$= \frac{1}{2}c + d - e$

ii. in terms of a, b, and c.

$\overrightarrow{CF} = a - b - \frac{1}{2}c$

① Illustrate the resultant vectors of $3\vec{a}+\vec{b}$ and $2\vec{b}-\vec{a}$ and label.





Heera Rajavel,
Zachary Zhang,
Ibrahim Naguthanawala

a) find the vector $\vec{AE}$ and $\vec{AF}$

$\vec{AE} = 5j + 3k$

$\vec{AF} = 5j - 4i + 3k$

b) Find the angle between the two diagonals

$\vec{AE} \cdot \vec{AF} = 34$

$|AE| \cdot |AF| = \sqrt{34} \cdot \sqrt{50} = 10\sqrt{17}$

$\cos\theta = \frac{34}{10\sqrt{17}}$

$\theta = \arccos\left(\frac{22}{10\sqrt{17}}\right) = 34°$

2.) Find a and b if $K(2,-2,0)$, $L(6,-5,2)$ and $M(a,3,b)$ are colinear

$\vec{KL} = t \cdot \vec{LM}$

① $\begin{pmatrix} 6-2 \\ -5-(-2) \\ 2-0 \end{pmatrix} = t \cdot \begin{pmatrix} a-6 \\ 3-(-5) \\ b-2 \end{pmatrix} \rightarrow$ $4 = t(a-6)$; $-3 = t \cdot -8 \rightarrow t = 3/8$; $2 = t\cdot(b-2)$

② $4 = 3/8\,(a-6)$ $2 = 3/8\,(b-2)$

$32/3 = a-6$ $16/3 = b-2$

1) find the velocity vector of a rocket moving in the direction $3i + 2j$ with a speed of 6km/hr.

$$\sqrt{3^2+2^2} = \sqrt{13} \Rightarrow \frac{3i+2j}{\sqrt{13}} = \text{unit vector}$$

$$6\left[\frac{3i+2j}{\sqrt{13}}\right] = \boxed{\frac{18i+12j}{\sqrt{13}}}$$

2) Find a if $p = 7i + 8j$ and $q = ai - 10j$ if they are perpendicular

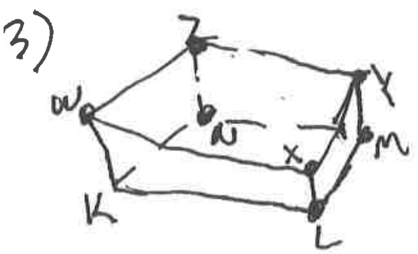
$p \cdot q = 0$

$7 \cdot a$

$8 \cdot {}^{-}10 \Rightarrow 7a - 80 = 0$

$$a = \frac{80}{7}$$

3)



$KL = 9 \quad LM = 6 \quad LX = 4$

Find $Y\hat{N}X$

Assume N is origin so $\begin{pmatrix}0\\0\\0\end{pmatrix}$

$Y = \begin{pmatrix}0\\9\\4\end{pmatrix}$

$X = \begin{pmatrix}6\\9\\4\end{pmatrix}$

$\overrightarrow{NY} = \begin{pmatrix}0\\9\\4\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\9\\4\end{pmatrix} \quad \overrightarrow{NX} = \begin{pmatrix}6\\9\\4\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}6\\9\\4\end{pmatrix}$

$\cos\theta = \frac{a \cdot b}{|a||b|} \qquad \overrightarrow{NY} \cdot \overrightarrow{NX} = \begin{pmatrix}0\\9\\4\end{pmatrix} \cdot \begin{pmatrix}6\\9\\4\end{pmatrix} = 0 + 81 + 16 = 97$

$|\overrightarrow{NY}| = \sqrt{0^2+9^2+4^2} = \sqrt{97} \qquad |\overrightarrow{NX}| = \sqrt{6^2+9^2+4^2} = \sqrt{133}$

$\cos\theta = \frac{97}{\sqrt{97}\sqrt{133}} \qquad \theta = \cos^{-1} \frac{97}{\sqrt{97}\sqrt{133}}$

CHAPTER 14:

Julia W., Akhil P., Erin P., Brayden A.

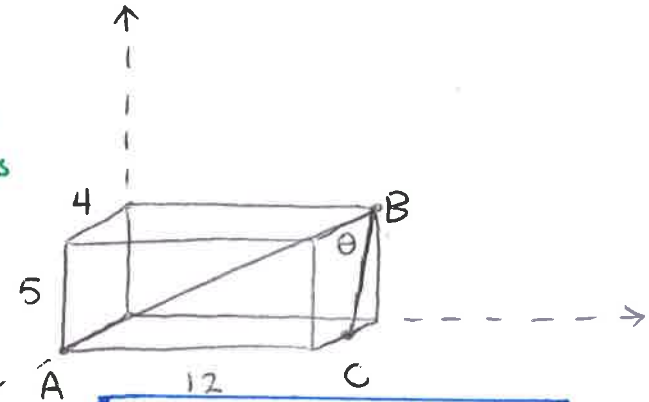
Per. 1

#1: Use vector methods to determine the exact measure of $A\hat{B}C$.

$A = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$ $\quad \overrightarrow{BA} = \begin{pmatrix} 4-0 \\ 0-12 \\ 0-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \\ -5 \end{pmatrix}$

+1 for correct points $\quad B = \begin{pmatrix} 0 \\ 12 \\ 5 \end{pmatrix}$ $\quad \overrightarrow{BC} = \begin{pmatrix} 2-0 \\ 12-12 \\ 0-5 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$ $\quad$ +1 for right vectors

$C = \begin{pmatrix} 2 \\ 12 \\ 0 \end{pmatrix}$



$$\cos\theta° = \frac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BC}|} = \frac{33}{\sqrt{185}\sqrt{29}} = \frac{33}{\sqrt{5365}} \Rightarrow \boxed{\theta° = \cos^{-1}\left(\frac{33}{\sqrt{5365}}\right)}$$

+1 for right equation $\quad$ +1 for correct values $\quad$ +1 for correct answer

$$\overrightarrow{BA} \bullet \overrightarrow{BC} = \begin{pmatrix} 4 \\ -12 \\ -5 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} = (4)(2) + (-12)(0) + (-5)(-5) = 33 \quad +1$$

$$|\overrightarrow{BA}| = \sqrt{4^2 + (-12)^2 + (-5)^2} = \sqrt{16 + 144 + 25} = \sqrt{185}$$

$$|\overrightarrow{BC}| = \sqrt{2^2 + 0^2 + (-5)^2} = \sqrt{4 + 0 + 25} = \sqrt{29}$$

#2: Given the vector $\overrightarrow{OP} = 7i - 5j + 6k$, find the following:

A) a vector in the opposite direction of $\overrightarrow{OP}$ with a magnitude of 3

unit vector $= \dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|}$ $\qquad$ Opposite direction $(\times -1)$

magnitude of 3 $(\times 3)$

$$|\overrightarrow{OP}| = \sqrt{7^2 + (-5)^2 + 6^2} = \sqrt{49 + 25 + 36} = \sqrt{110}$$ +1 for finding magnitude

$$\frac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \frac{7i - 5j + 6k}{\sqrt{110}} = \frac{7}{\sqrt{110}}i - \frac{5}{\sqrt{110}}j + \frac{6}{\sqrt{110}}k$$ +1 for unit vector

$$-3 \times \frac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \boxed{-\frac{21}{\sqrt{110}}i + \frac{15}{\sqrt{110}}j - \frac{18}{\sqrt{110}}k}$$ +1 for right answer

#3: Find values for a & b so D(a, 13, 6), E(7, 5, −3), and F(5, b, 0) are collinear.

$D = \begin{pmatrix} a \\ 13 \\ 6 \end{pmatrix}$

$E = \begin{pmatrix} 7 \\ 5 \\ -3 \end{pmatrix}$

$F = \begin{pmatrix} 5 \\ b \\ 0 \end{pmatrix}$

$\overrightarrow{DE} = \begin{pmatrix} 7-a \\ 5-13 \\ -3-6 \end{pmatrix} = \begin{pmatrix} 7-a \\ -8 \\ -9 \end{pmatrix}$

$\overrightarrow{EF} = \begin{pmatrix} 5-7 \\ b-5 \\ 0-(-3) \end{pmatrix} = \begin{pmatrix} -2 \\ b-5 \\ 3 \end{pmatrix}$

$\overrightarrow{DE} = k(\overrightarrow{EF})$

+1 for right setup

$7-a = -2k$

$7-a = -2(-3)$

$7-a = 6$

$\boxed{a = 1}$

+½ for this answer

$-8 = k(b-5)$

$-8 = -3(b-5)$

$-8 = -3b + 15$

$-23 = -3b$

$\boxed{b = \frac{23}{3}}$

+½ for this answer

$-9 = 3k$

$k = -3$