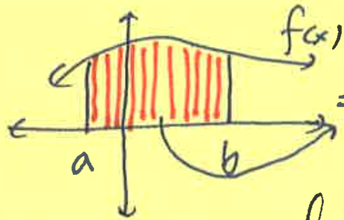
IB Math HL1 21J.3 Properties of Definite Integrals



$\Rightarrow \int_a^b f(x)\,dx$

$\lim_{n\to\infty} \sum_{k=0}^{n} f(a+\Delta x k)\Delta x$

$\Delta x = \frac{b-a}{n}.$

1. a. $\int_1^4 3x^2 dx =$

$= x^3 \Big|_{x=1}^{x=4}$

$= 4^3 - 1^3 = \boxed{63}$

b. $\int_4^1 3x^2 dx =$

$= x^3 \Big|_{x=4}^{x=1}$

$= 1^3 - 4^3 = -63$

① $\boxed{\int_b^a f(x)dx = -\int_a^b f(x)dx}$

2. $\int_1^4 15x^2 dx = \frac{15}{3}x^3 \Big|_{x=1}^{x=4}$

$= 5\left[4^3 - 1^3\right]$

$= 5 \cdot 63$

② $\boxed{\int_a^b cf(x)dx = c\int_a^b f(x)dx}$

3. $\int_1^2 3x^2 dx + \int_2^4 3x^2 dx = \int_1^4 3x^2 dx.$

③ $\boxed{\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx}$

4. a. $\int_1^4 3x^2 dx + \int_1^4 2x dx =$

$= \int_1^4 (3x^2 + 2x)dx$

b. $\int_1^4 \left(3x^2 + 2x\right)dx = \int_1^4 3x^2 dx + \int_1^4 2x dx$

④ $\boxed{\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx}$

5. Given $\int_{-2}^{5} f(x)dx = 12$, $\int_{-2}^{1} f(x)dx = -2$, $\int_{-2}^{5} g(x)dx = 7$, find each of the following.

a. $\int_{1}^{5} f(x)dx = \int_{-2}^{5} f(x)dx - \int_{-2}^{1} f(x)dx$

$= 12 - (-2) = \boxed{14}$

b. $\int_{5}^{1} f(x)dx = \boxed{-14}$

c. $\int_{-2}^{5} 3f(x)dx$

$= 3 \cdot 12 = \boxed{36}$

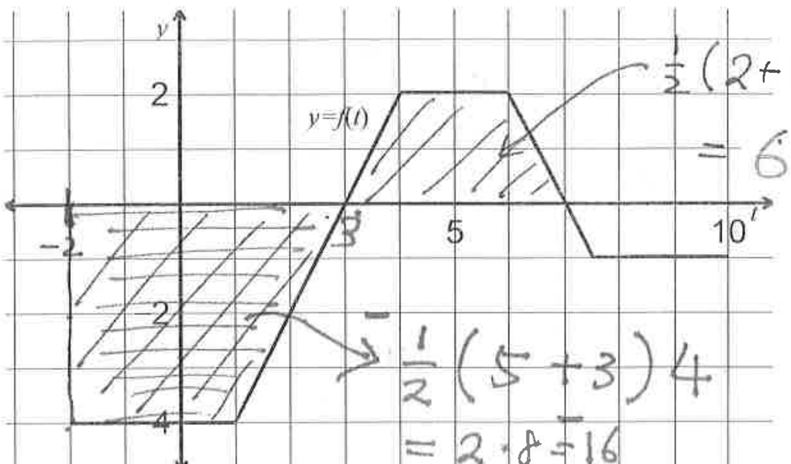
d. $\int_{-2}^{5} \left(3f(x) - 2g(x)\right)dx = 3 \cdot 12 - 2 \cdot 7$

$= \boxed{22}$

6. Given is the graph of $y = f(t)$.

Another function is defined as $g(x) = \int_{-2}^{x} f(t)dt, \ -2 \le x \le 10$.

Find the following.



$\frac{1}{2}(2+4)2$

$= 6$

$\frac{1}{2}(5+3)4$

$= 2 \cdot 8 = 16$

a. $g(-2) =$ ________

$= \int_{-2}^{-2} f(t)dt = 0$

b. $g(7) =$ $-16 + 6 = \boxed{-10}$

$= \int_{-2}^{7} f(t)dt$

7. The graph of the function $y = f(x)$ consists of two semicircles.

Find the exact value of the following integral:

$\int_{-2}^{3} f(x)dx = 4 - \pi + 6 + \frac{9}{4}\pi$

$= 10 - \frac{5}{4}\pi$

$= 4 - \frac{1}{4}(\pi \cdot 2^2)$

$= 4 - \pi$

$2 = 2 \cdot 3 = 6$

3

$3 = \frac{1}{4}(\pi \cdot 3^2)$

3

$= \frac{9}{4}\pi$