Day two : Divisibility

Math Induction proof process for a given conjecture (statement):
Step 1: Show that the statement is true for an initial case, n=1.
Step 2: Assume that the statement is true for n=k where $k \in Z^+$.
Step 3: Prove that the statement is true for n=k+1.
Step 4: ∴ The statement is true for $n \in Z^+$

Problem 1)

Using Mathematical induction, prove that $P(n): n^3 + 2n$ is divisible by 3 true for any $n \in \mathbb{Z}^+$.

1). When $n=1$, $p(1) = 1^3 + 2\cdot n = 3 \Rightarrow 3 = 3\cdot 1$, $P(n)$ is divisible by 3.

2) When $n=k$, Assume $p(n)$ is divisible by 3

$k^3 + 2k = 3\cdot A$ where $A \in Z^+$

or $k(k^2+2) = 3\cdot A$

3) If $n = k+1$, $(k+1)^3 + 2(k+1)$

$= k^3 + 3k^2 + 3k + 1 + 2k + 2$

$= (k^3 + 2k) + 3k^2 + 3k + 3$ ($k^3+2k = 3\cdot A$ from step 2)

$= 3\cdot A + 3(k^2+k+1)$

4) ∴ $p(n)$ is divisible by 3 for $n \in Z^+$

Problem 2)

Using Mathematical induction, prove $5^n - 1$ is evenly divisible by 4 for all $n \geq 1$.

1) When $n=1$, $5-1 = 4 = 4\cdot 1$, $5^n - 1$ is divisible by 4 for $n=1$.

2) When $n=k$, Assume 5^n-1 is divisible by 4.

$5^n - 1 = 4\cdot A$ where $A \in Z^+$

3) If $n = k+1$, $5^{k+1} - 1$

$= 5\cdot 5^k - 1$

$= 5\cdot 5^k - 5 + 4$

$= 5(5^k - 1) + 4$

$= 5(4\cdot A) + 4$

$20\cdot A + 4$

$= 4[5A+1]$

4) ∴ $5^n - 1$ is divisible by 4 for all $n \geq 1$.

IB Questions) Do on separate paper

1.

Use the method of mathematical induction to prove that $5^{2n}-24n-1$ is divisible by 576 for $n \in \mathbb{Z}^+$.

2.

Prove, by mathematical induction, that $7^{8n+3}+2$, $n \in \mathbb{N}$, is divisible by 5.

3.

Using Mathematical induction, prove $10^n + 3\cdot 4^{n+2} + 5$ is evenly divisible by 9 for all $n \geq 1$.

#1.

1) When $n=1$

$5^2 - 24 - 1 = 0$

$0 = 576 \cdot 0$

So the statement is true for $n=1$

2) when $n=k$

Assume $5^{2k} - 24k - 1 = 576 \cdot A$ where $A \in \mathbb{Z}^+$

3) If $n = k+1$

$5^{2(k+1)} - 24(k+1) - 1$

$= 25 \cdot 5^{2k} - 24k - 24 - 1$

$= 25\ 5^{2k} - 25(24k) - 25 + 24(24k)$

$= 25[5^{2k} - 24k - 1] + 576k$.

$= 25 \cdot [576 \cdot A] + 576k$

4) $\therefore$ $5^{2n} - 24n - 1$ is divisible by 576 for $n \in \mathbb{Z}^+$

#2.

36
8
49
$+\ 49$
441
206
2401

1) when $n=1$

$7^{8+3} + 2 = 7^2 \cdot 7^2 \cdot 7^2 \cdot 7^2 \cdot 7^3 + 2$

$= \underbrace{49 \cdot 49}\cdot \underbrace{49 \cdot 49} \cdot 49 \cdot 7$

$= (\ldots 1)(\ldots 3) + 2 = \ldots 5$ Always 5 as last digit.

2) when $n=k$

Assume $7^{8k+3} + 2 = 5 \cdot A$ where $A \in \mathbb{Z}^+$

3) If $n = k+1$

$7^{8(k+1)+3} + 2$

$= 7^{(8k+8+3)} + 2$

$= 7^8[7^{8k+3}] + 2$

$= \underbrace{7^8\,[7^8 \cdot 7^k \cdot 7^3]} + 2 \Rightarrow \ldots 3 + 2 = \ldots 5$

$\Rightarrow (7^8) \cdot 7^k \cdot 7^3 + 2$ will always have the ones digit 5.

one's digit is 3. $\Rightarrow$ will have one's digit 5.

$\therefore$ $7^{8n+3} + 2$ is always divisible by 5 for $n \in \mathbb{N}$.

#3. $E(n)$: $10^n + 3 \cdot 4^{n+2} + 5$

1) when $n=1$ $10 + 3 \cdot 4^3 + 5 = 20719$ is divisible by 9.

2) when $n=k$ $\Rightarrow$ Assume $10^k + 3 \cdot 4^{k+2} + 5 = 9 \cdot m$ where $m \in \mathbb{Z}^+$.

3) If $n = k+1$ $\Rightarrow$ $10^{k+1} + 3 \cdot 4^{k+3} + 5 = 10 \cdot 10^k + 4 \cdot 3 \cdot 4^{k+1} + 5$

$= 10(10^k + 3 \cdot 4^{k+2} + 5) - 18 \cdot 4^{k+2} - 45$

$= (10)(9 \cdot m) - 9(2 \cdot 4^{k+2} + 5)$

4) $\therefore$ $E(n)$ is divisible by 9 for $n \geq 1$.