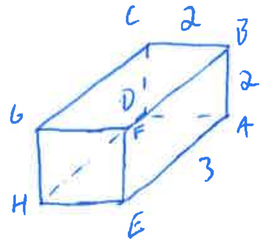
1) 


Find vectors $\vec{AG}$ and $\vec{EC}$ and then the angle between them.

$\vec{AG}\begin{pmatrix}3\\-2\\2\end{pmatrix}$ $\vec{EC}\begin{pmatrix}-3\\-2\\2\end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix}3\\-2\\2\end{pmatrix}\cdot\begin{pmatrix}-3\\-2\\2\end{pmatrix}}{|AG||EC|} = -\dfrac{1}{17}$

$\theta = \arccos\left(-\frac{1}{17}\right)$ $\theta = 93.4^\circ$ or 1.63 radians

2) Find values of a and b so that R(a+2, 3, 7), Q(6, 5, 1) and W(4, b, 9) are collinear.

$\vec{RT} = \begin{pmatrix}6-a+2\\5-3\\1-7\end{pmatrix} = \begin{pmatrix}8-a\\2\\-6\end{pmatrix}$ $\vec{QW} = \begin{pmatrix}4-6\\b-5\\9-1\end{pmatrix} = \begin{pmatrix}-2\\b-5\\8\end{pmatrix}$

$$\begin{pmatrix}8-a\\2\\-6\end{pmatrix} = t\begin{pmatrix}-2\\b-5\\8\end{pmatrix}$$

$8-a = -2t$
$2 = t(b-5)$
$-6 = 8t$

$t = -6/8 = -\frac{3}{4}$

$8-a = +\cancel{2}\left(+\frac{3}{\cancel{4}}\right)$
$8-a = \frac{3}{2}$
$-a = \frac{19}{2}$
$\boxed{a = -\frac{19}{2}}$

$2 = -\frac{3}{4}(b-5)$
$8 = -3b+15$
$-7 = -3b$
$\boxed{b = \frac{7}{3}}$

3) Find the angle between the vectors $\begin{pmatrix}3\\7\\-5\end{pmatrix}$ and $\begin{pmatrix}1\\-4\\2\end{pmatrix}$

$\cos\theta = \dfrac{a\cdot b}{|a||b|}$

$a\cdot b = 3-28-10 = -35$

Magnitude: $\sqrt{3^2+7^2+5^2} = \sqrt{83}$ $\sqrt{1^2+4^2+2^2} = \sqrt{21}$

$\theta = \cos^{-1}\left(\dfrac{-35}{\sqrt{83}\sqrt{21}}\right) \approx \theta = 147^\circ$

Emory J, Flavia M, Kevin X, Brian C

1) Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ if $A(7,8,0)$, $B(10,5,2)$ and $C(3,1,2)$.

Solution:

$$\overrightarrow{AB} = \begin{pmatrix} 10-7 \\ 5-8 \\ 2-0 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} 3-10 \\ 1-5 \\ 2-2 \end{pmatrix} = \begin{pmatrix} -7 \\ -4 \\ 0 \end{pmatrix}$$

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|\,|\vec{w}|}$$

$$\cos\theta = \frac{(3\cdot -7)+(-3\cdot -4)+(2\cdot 0)}{\sqrt{3^2+(-3)^2+2^2}\ \sqrt{(-7)^2+(-4)^2+0^2}}$$

$$= \frac{-21+12+0}{\sqrt{9+9+4}\ \sqrt{49+16}} = \frac{9}{\sqrt{22}\ \sqrt{65}}$$

$$\cos^{-1}\left(\frac{9}{\sqrt{22}\ \sqrt{65}}\right) = \boxed{\theta \approx 76.2^\circ}$$

2) Suppose $\vec{a} = \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 5 \\ -2 \\ -6 \end{pmatrix}$.

a) Find $x$ such that $a - b = 3x$.

$$\begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ -6 \end{pmatrix} = 3\underbrace{\begin{pmatrix} i \\ j \\ k \end{pmatrix}}_{\vec{x}}$$

$$5-5 = 3i \qquad 3-(-2) = 3j \qquad -1-(-6) = 3k$$

$$0 = 3i \qquad 5 = 3j \qquad -1+6 = 3k$$

$$\boxed{i=0} \qquad \boxed{j = \frac{5}{3}} \qquad 5 = 3k \qquad \boxed{k = \frac{5}{3}}$$

$$\therefore x = \begin{pmatrix} 0 \\ 5/3 \\ 5/3 \end{pmatrix}$$

b) Find constants $p$ and $q$ if $\vec{c} = \begin{pmatrix} p \\ -2 \\ q \end{pmatrix}$ is parallel to $\vec{a}$.

$$\vec{c} = t\vec{a} \Rightarrow \begin{pmatrix} 5 \\ 3 \\ -1 \end{pmatrix} = t\begin{pmatrix} p \\ -2 \\ q \end{pmatrix}$$

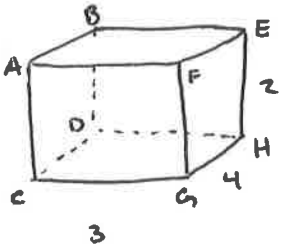
$$3 = -2t$$

$$t = -\frac{3}{2}$$

$$5 = \left(-\frac{3}{2}\right)p \qquad -1 = \left(-\frac{3}{2}\right)q$$

$$\boxed{p = -\frac{10}{3}} \qquad \boxed{q = \frac{2}{3}}$$

3)




What is $\angle ABG$?

$\overrightarrow{BA} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$ $\quad \overrightarrow{BG} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$

$\cos\theta = \dfrac{\overrightarrow{BA} \cdot \overrightarrow{BG}}{|\overrightarrow{BA}||\overrightarrow{BG}|}$

$\cos\theta = \dfrac{16}{4\sqrt{29}}$

$\boxed{\theta = 42.0°}$

$\overrightarrow{BA} \cdot \overrightarrow{BG} = 16 + 0 + 0 = 16$

$|\overrightarrow{BA}| = \sqrt{4^2 + 0^2 + 0^2} = \sqrt{16} = 4$

$|\overrightarrow{BG}| = \sqrt{4^2 + 3^2 + (-2)^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

Vincent, Ben, Rutuja, Thevina    Ch14 Quiz Questions

1. Find v if $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ v \end{bmatrix}$ is a unit vector

$$\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + v^2} = 1$$

$$\sqrt{\frac{3}{4} + v^2} = 1$$

$$v^2 = 1 - \frac{3}{4}$$

$$v^2 = \frac{1}{4}$$

$$v = \pm \frac{1}{2}$$

2. Find values of a and b so that P(4, a, 0), Q(10, 12, 6), R(13, 5, b) are collinear

$$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ 12-a \\ 6 \end{pmatrix} \quad \overrightarrow{QR} = \begin{pmatrix} 3 \\ -7 \\ b-6 \end{pmatrix}$$

① $6 = 3t$    $t = 2$

② $12 - a = -7t$    $12 - a = -14$    $a = 26$

③ $6 = t(b-6)$    $3 = b - 6$    $b = 9$

3. given P(1, 7, 10) and Q(-8, 5, 13)

a.) Find $\overrightarrow{QP}$

$$\begin{pmatrix} 9 \\ 2 \\ -3 \end{pmatrix}$$

b) find $\left|\frac{1}{4}\overrightarrow{PQ}\right|$

$$\overrightarrow{PQ} = \begin{pmatrix} -9 \\ -2 \\ 3 \end{pmatrix} \quad \frac{1}{4}PQ = \begin{matrix} -\frac{9}{4} \\ -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}$$

$$\left|\frac{1}{4}PQ\right| = \sqrt{\frac{81}{16} + \frac{1}{4} + \frac{9}{16}}$$

$$= \sqrt{\frac{47}{8}} = \frac{\sqrt{47}}{2\sqrt{2}}$$

Angeline Yu, Anika Ajwani, Dhruv Jagannath, Roger Wang

1) given $\vec{a} = 2i - 7j$ and $\vec{b} = i + 6j$

a) find $\vec{a} + 2\vec{b}$

S: $2i - 7j + 2i + 12j$

$= \boxed{4i + 4j}$

b) find $2\vec{b} - \vec{a}$

S: $2i + 12j - 2i + 7j$

$\boxed{19j}$

2) find a if $p = 4i + j$ and $q = ai + 6j$ and its perpendicular

$p \cdot q = 0 = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} a \\ 6 \end{pmatrix}$

$4a + 6 = 0 \rightarrow 4a = -6$

$\boxed{a = \frac{3}{2}}$

3) find a and b if $A(5,-5,1)$ $B(10,2,3)$
$C(a,0,b)$ are collinear

$\vec{AB} = t\,\vec{BC}$

$$\begin{pmatrix}10\\2\\3\end{pmatrix} - \begin{pmatrix}5\\-5\\1\end{pmatrix} = \begin{pmatrix}5\\7\\2\end{pmatrix}$$

$$\begin{pmatrix}a\\0\\b\end{pmatrix} - \begin{pmatrix}10\\2\\3\end{pmatrix} = \begin{matrix}a-10\\-2\\b-3\end{matrix}$$

$$\begin{pmatrix}5\\7\\2\end{pmatrix} = t\begin{pmatrix}a-10\\-2\\b-3\end{pmatrix}$$

$t = -\frac{7}{2}$

$5 = -\frac{7}{2}(a-10)$

$2 = -\frac{7}{2}(b-3)$

$\frac{-10}{35} = a-10 \qquad \frac{-4}{14} = b-3 \qquad \frac{-10}{35} + \frac{350}{35} = a$

$\frac{-4}{14} + \frac{42}{14} = b \qquad a = \frac{340}{35} = \boxed{\frac{68}{7}}$

$b = \frac{38}{14} = \boxed{\frac{16}{7}}$

Blake Harrison, Troy Lu, Matthew Hung, Enya Song

1) Find $w$ if $3i + wj + (w+2)k$ and $i + 2j + wk$ are perpendicular

$$\cos 90 = \frac{A \cdot B}{|A| \cdot |B|}$$

$$0 = \frac{0}{\cancel{|A| \cdot |B|}}$$

$$A \cdot B = 0$$

$$\begin{pmatrix} 3 \\ w \\ w+2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ w \end{pmatrix}$$

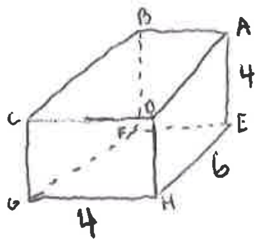
$$w^2 + 2w + 2w + 3$$

$$w^2 + 4w + 3$$

$$(w+3)(w+1) = 0$$

$$\boxed{\begin{matrix} w = -1 \\ w = -3 \end{matrix}}$$

2)




Find the angle between diagonals $\vec{GA}$ and $\vec{CE}$

$$\vec{GA} = \begin{pmatrix} 4 \\ -6 \\ 4 \end{pmatrix} \quad \vec{CE} = \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 4 \\ -6 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -6 \\ -4 \end{pmatrix}$$

$$\cancel{16} + 36 - \cancel{16} = \boxed{36 = A \cdot B}$$

$$|A| = \sqrt{4^2 + (-6)^2 + 4^2} = \sqrt{68}$$

$$|B| = \sqrt{4^2 + (-6)^2 + (-4)^2} = \sqrt{68}$$

$$\cos \theta = \frac{A \cdot B}{|A| \cdot |B|}$$

$$\cos \theta = \frac{36}{\sqrt{68} \cdot \sqrt{68}}$$

$$\cos \theta = \frac{36}{68}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{9}{17}\right)}$$

$$\theta \approx 58.0°$$

3) Given $\vec{c} = 36i - 4k$ and $\vec{f} = 25j + 4k$

a) Find $2\vec{c} - \vec{f}$

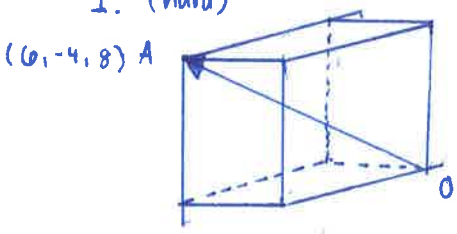
$$2\begin{pmatrix} 36 \\ 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 25 \\ 4 \end{pmatrix} = \begin{pmatrix} 72 \\ 0 \\ -8 \end{pmatrix} - \begin{pmatrix} 0 \\ 25 \\ 4 \end{pmatrix} = \begin{pmatrix} 72 \\ -25 \\ -12 \end{pmatrix} = \boxed{72i - 25j - 12k}$$

b) Find $10\vec{f} - \vec{c}$

$$10\begin{pmatrix} 0 \\ 25 \\ 4 \end{pmatrix} - \begin{pmatrix} 36 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 250 \\ 40 \end{pmatrix} - \begin{pmatrix} 36 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} -36 \\ 250 \\ 44 \end{pmatrix} = \boxed{-36i + 250j + 44k}$$

# QUIZ PROBLEMS

1. (hard)




Find the angle between
a) x-axis
b) y-axis
c) z-axis

ANSWERS:

$\sqrt{6^2+(-4)^2+(8)^2}$

a) $\cos\theta = \left(\frac{6}{\sqrt{116}}\right)$

$\theta = \arccos\left(\frac{6}{\sqrt{116}}\right)$

b) $\cos\theta = \left(\frac{-4}{\sqrt{116}}\right)$

$\theta = \arccos\left(\frac{-4}{\sqrt{116}}\right)$

c) $\cos\theta = \left(\frac{8}{\sqrt{116}}\right)$

$\theta = \arccos\left(\frac{8}{\sqrt{116}}\right)$

2.) If v is defined by $\begin{pmatrix}3\\-4\\2\end{pmatrix}$ and f is defined as $\begin{pmatrix}7\\2\\-1\end{pmatrix}$, then:

a.) Find the magnitude of $\vec{VF}$

$|\vec{VF}| = \sqrt{(3-7)^2+(-4-2)^2+(2-(-1))^2}$

$|\vec{VF}| = \sqrt{16+36+9}$

$\boxed{|\vec{VF}| = \sqrt{61}}$

b.) Find the dot product of V and F

$\vec{V}\cdot\vec{F} = (3)(7)+(-4)(2)+(2)(-1)$

$\vec{V}\cdot\vec{F} = 21-8-2$

$\boxed{\vec{V}\cdot\vec{F} = 11}$

3. Find all values of t if vectors $\vec{AB}$ and $\vec{CD}$ are perpendicular, given vector $\vec{AB} = 8i + (2-t)j + (6-t)k$ and vector $\vec{CD} = (36-t)i + (28-t)j + k$

$\vec{AB}$ $\begin{pmatrix}8\\2-t\\6-t\end{pmatrix}$ $\vec{CD}$ $\begin{pmatrix}36-t\\28-t\\1\end{pmatrix}$

perpendicular angle is 90

$\cos\theta = 0$

$\cos\theta = \frac{a\cdot b}{|a||b|}$

$\cos\theta = 0$

$0 = \frac{a\cdot b}{|a||b|}$

$0 = a\cdot b$

$8(36-t) + (2-t)(28-t) + (6-t)(1) = 0$

$288 - 8t + (56-28t-2t+t^2) + (6-t) = 0$

$t^2 - 39t + 350 = 0$

$(t-25)(t-14) = 0$ $t = 25$ or $t = 14$

1. Given $\vec{v} = 3i - 7j$ and $\vec{w} = 4i + 6j$

a) Find $\vec{x}$ such that $\vec{v} + 2\vec{x} = \vec{w}$ → $3i - 7j + 2x = 4i + 6j$ → $2x = i + 13j$ → $\boxed{x = \frac{1}{2}i + \frac{13}{2}j}$

b) Find $p$ if $\vec{u} = pi + 21j$ is parallel to $\vec{v}$ → $\frac{p}{3} = \frac{21}{-7}$ → $-7p = 63$ → $\boxed{p = -9}$

c) Find $q$ if $\vec{r} = qi + q^2 j$ is perpendicular to $\vec{w}$ → $(q)(4) + (q^2)(6) = 0$ → $6q^2 + 4q = 0$ → $(6q+4)(q) = (0)$ → $\boxed{q = -\frac{2}{3}}$

2. Given $A(a^2, 6, 10-b)$, $B(7, e, 8)$, $C(4, 3, 2)$ and $D(3, \frac{1}{4}d, 0)$ are colinear, find $a, b, d, e$.

$7 - a^2 = t(4-7)$ → $7 - a^2 = 3(-3)$ $\quad a^2 = 16$ $\quad \boxed{a = \pm 4}$

$e - 6 = t(3-c)$ → $e - 6 = 3(3-c)$ $\quad c - 6 = 9 - 3c$ $\quad 4c = 15$ $\quad \boxed{c = \frac{15}{4}}$

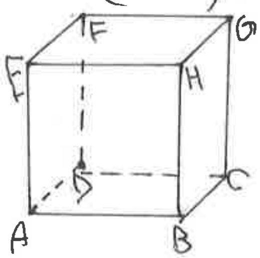
$b - 2 = t(2-8)$ → $b - 2 = 3(-6)$ $\quad \boxed{b = -16}$

$3 - e = t(\frac{1}{4}d - 3)$ → $3 - (\frac{15}{4}) = 3(\frac{1}{4}d - 3)$ $\quad -\frac{3}{4} = \frac{3}{4}d - 9$ $\quad -\frac{1}{4} = \frac{1}{4}d - 3$

$\boxed{d = 11}$

$2 - 8 = t(0-2)$ → $-6 = 2t$ $\quad t = 3$

3.




Given the cube with side lengths of 3 cm

a. what is the angle between $\overrightarrow{DC}$ and $\overrightarrow{FB}$ $\qquad$ a. = $\theta = 54.7°$

b. What is the magnitude of $\overrightarrow{DH}$ $\qquad$ b. = $3\sqrt{3}$

c. what is the dot product of $\overrightarrow{AG}$ and $\overrightarrow{BC}$ $\qquad$ c. = 9 cm

1. Find $a$ if $g = \begin{pmatrix} a \\ 14 \end{pmatrix}$ and $h = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$ are perpendicular.

$$\cos 90° = \frac{g \cdot h}{|g| \times |h|}$$

$$8a + 14 = 0$$

$$a = -\frac{14}{8} = \boxed{-\frac{7}{4}}$$

2. $a = \begin{pmatrix} 5 \\ 5 \\ t \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ s \\ r \end{pmatrix}$, and $c = \begin{pmatrix} 5 \\ t \\ 5 \end{pmatrix}$ are mutually perpendicular. Find $t$, $r$, and $s$.

$$a \cdot c = 0$$

$$a \cdot b = 0$$

$$c \cdot b = 0$$

$$\left.\begin{aligned} 25 + 5t + 5t &= 0 \\ 30 + 5s + tr &= 0 \\ 30 + ts + 5r &= 0 \end{aligned}\right\} \text{system of equations}$$

$$\boxed{t = -2.5}$$

$$30 + 5s - 2.5r = 30 - 2.5s + 5r$$

$$r = s$$

$$30 - 2.5s + 5s = 0$$

$$30 + 2.5s = 0$$

$$\boxed{r = -12,\ s = -12}$$

Queenie Nguyen Gabriel Rishi Ocean Rockwell Devin Seiu

3. Find the angle between $c = \begin{pmatrix} 7 \\ 14 \\ 6 \end{pmatrix}$ and $d = \begin{pmatrix} 3 \\ 5 \\ 9 \end{pmatrix}$.

$$\theta = \arccos\left(\frac{c \cdot d}{|c|\,|d|}\right)$$

$$c \cdot d = 21 + 70 + 54 = 165$$

$$|c| = \sqrt{49 + 196 + 36} = \sqrt{281}$$

$$|d| = \sqrt{9 + 25 + 81} = \sqrt{115}$$

$$\theta = \arccos\left(\frac{165}{\sqrt{281} \times \sqrt{115}}\right) \approx \boxed{23.4^\circ}$$