practice !

1) Find $\frac{dy}{dx}$, $2x^2 - 3y^2 = 2$

2) Find the Equation of normal to the curve to $2x^2 - 3y^2 = 2$ at $x = 5$.

3) A kite 60 m high is being carried away by a wind at 4 m/s.
How fast is the string being let out when the string is 100 m long ?

4) Find a particular solution for $\frac{dy}{dx} = \frac{y}{x^2+1}$ and $y(0) = e^2$

5) $a = 5s + 3$, Find v in terms of s
Given $s = 2$ when $v = 8 \frac{m}{s}$.

6) Find the general solution for $\frac{dy}{dx} + y\tan x = \cos^2 x$

7) Find the general solution for $(x^2+y^2)dx + 2xy\,dy = 0$

1. $2x^2-3y^2=2$

$\frac{d}{dx}(2x^2-3y^2)=\frac{d}{dx}(2)$

$4x-6y\cdot\frac{dy}{dx}=0$

$-6y\cdot\frac{dy}{dx}=-4x$

$\frac{dy}{dx}=\frac{-4x}{-6y}$

$\boxed{\frac{dy}{dx}=\frac{2x}{3y}}$

2. $2x^2-3y^2=2 \leftarrow x=5$

$2(5)^2-3y^2=2$

$50-3y^2=2$

$-3y^2=-48$

$y^2=16$

$y=\pm 4$

Slope of normal line: $-\frac{1}{\frac{dy}{dx}}$

$=-\frac{1}{\frac{2x}{3y}}$ (from last problem)

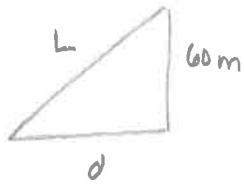
$=-\frac{3y}{2x}$

$(5,4): \frac{-3(4)}{2(5)}=-\frac{6}{5}$

$(5,-4): \frac{-3(-4)}{2(5)}=\frac{6}{5}$

$$\boxed{\begin{aligned} y-4&=-\tfrac{6}{5}(x-5)\\ y+4&=\tfrac{6}{5}(x-5)\end{aligned}}$$

3.



$L=100\text{m}$

$D=\sqrt{100^2-60^2}$

$=80\text{m}$

$\frac{dD}{dt}=4\text{ m/s}$

$\frac{dL}{dt}=?$

$(60)^2+D^2=L^2$

$3600+D^2=L^2$

$\frac{d}{dt}(3600+D^2)=\frac{d}{dt}(L^2)$

$2D\cdot\frac{dD}{dt}=2L\cdot\frac{dL}{dt}$

$2\cdot 80\cdot 4=2\cdot 100\cdot\frac{dL}{dt}$

$\frac{640}{200}=\frac{dL}{dt}$

$\boxed{\frac{dL}{dt}=3.2\text{ m/s}}$

4. $\frac{dy}{dx}=\frac{y}{x^2+1}$

$\frac{dy}{y}=\frac{dx}{x^2+1}$

$\int\frac{dy}{y}=\int\frac{dx}{x^2+1}$

$\ln y=\arctan x+C$

$e^{\ln y}=e^{\arctan x+C}$ $(e^C=A)$

$y=Ae^{\arctan x}$

$(y=e^2,\ x=0)$

$e^2=A\cdot e^0$

$e^2=A$

$\boxed{y=e^{\arctan x+2}}$

5. $a=5s+3$

$a=\frac{dv}{ds}\cdot\frac{ds}{dt}=\frac{dv}{ds}\cdot v$

$5s+3=\frac{dv}{ds}\cdot v$

$(5s+3)\,ds=v\,dv$

$\int(5s+3)\,ds=\int v\,dv$

$\frac{5}{2}s^2+3s+C=\frac{1}{2}v^2$

$(s=2,\ v=8)$

$\frac{5}{2}(2)^2+3(2)+C=\frac{1}{2}(8)^2$

$10+6+C=32$

$C=16$

$\frac{5}{2}s^2+3s+16=\frac{1}{2}v^2$

$5s^2+6s+32=v^2$

$\boxed{v=\sqrt{5s^2+6s+32}}$

6. $y'+y\cdot\tan x=\cos^2 x$

$I=e^{\int p\,dx}=e^{\int\tan x\,dx}=e^{-\ln|\cos x|}=-\cos x$

$-\cos x\,(y'+\tan x\cdot y)=-\cos x\cdot\cos^2 x$

$-\cos x\cdot y'-\sin x\cdot y=-\cos^3 x$

$\int d[-\cos x\cdot y]=\int-\cos^3 x\,dx$

$-\cos x\cdot y=\int-\cos^3 x\,dx$

$-\cos x\cdot y=-\sin x+\frac{1}{3}\sin^3 x+C$

$\boxed{y=\tan x-\frac{1}{3}\sin^2 x\tan x-\frac{C}{\cos x}}$

7. $(x^2+y^2)\,dx + 2xy\,dy = 0$

$$(x^2+y^2) + 2xy\cdot\frac{dy}{dx} = 0$$

$$x^2$$

$$1 + \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right)\cdot\frac{dy}{dx} = 0$$

$$v = \frac{y}{x}\ , \quad v + x\cdot\frac{dv}{dx} = \frac{dy}{dx}$$

$$1 + v^2 + 2v\left(v + x\cdot\frac{dv}{dx}\right) = 0$$

$$1 + v^2 + 2v^2 + 2vx\cdot\frac{dv}{dx} = 0$$

$$2vx\cdot\frac{dv}{dx} = -3v^2 - 1$$

$$x\cdot\frac{dv}{dx} = \frac{-3v^2-1}{2v}$$

$$\frac{x}{dx} = \frac{-3v^2-1}{2v\,dv}$$

$$\int\frac{dx}{x} = \int\frac{2v\,dv}{-3v^2-1}$$

$$\ln x = -\frac{1}{3}\ln|-3v^2-1| + c$$

$$x = e^{\ln(-3v^2-1)^{-\frac{1}{3}} + c}$$

$$(e^c = A)$$

$$x = A(-3v^2-1)^{-\frac{1}{3}} \qquad \left(v = \frac{y}{x}\right)$$

$$x = A\left(-3\left(\frac{y}{x}\right)^2 - 1\right)^{-\frac{1}{3}}$$

$$\left(\frac{x}{A}\right)^{-3} = \left(\left(-3\left(\frac{y}{x}\right)^2 - 1\right)^{-\frac{1}{3}}\right)^{-3}$$

$$\left(\frac{A}{x}\right)^3 = -3\left(\frac{y}{x}\right)^2 - 1$$

$$\left(\frac{A}{x}\right)^3 + 1 = -3\left(\frac{y}{x}\right)^2$$

$$-\frac{1}{3}\left(\left(\frac{A}{x}\right)^3 + 1\right) = \left(\frac{y}{x}\right)^2$$

$$\sqrt{-\frac{1}{3}\left(\left(\frac{A}{x}\right)^3 + 1\right)} = \frac{y}{x}$$

$$\boxed{y = x\sqrt{-\frac{1}{3}\left(\left(\frac{A}{x}\right)^3 + 1\right)}}$$