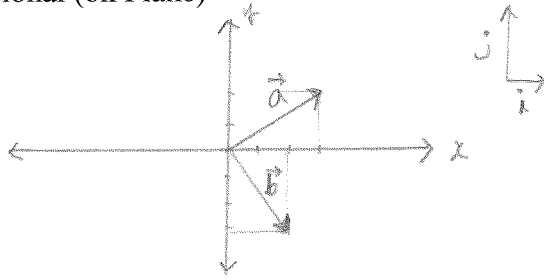


IB Math 2 : Vector Notes

Two-dimensional (on Plane)



• Basis vectors: $\vec{a} = 3\vec{i} + 2\vec{j}$ and $\vec{b} = 2\vec{i} - 3\vec{j}$

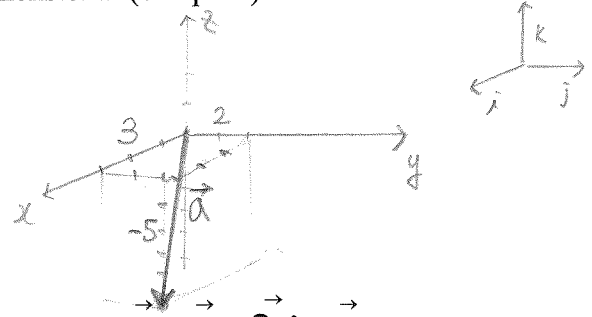
• Column Vectors: $\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

• Magnitude of a vector: $|\vec{a}| = \sqrt{3^2 + 2^2} = \sqrt{13}$

• Unit vector: $\frac{\vec{a}}{|\vec{a}|} = \frac{3\vec{i} + 2\vec{j}}{\sqrt{3^2 + 2^2}} = \frac{3\vec{i} + 2\vec{j}}{\sqrt{13}}$ OR $\frac{3}{\sqrt{13}}\vec{i} + \frac{2}{\sqrt{13}}\vec{j}$

key

Three-dimensional (on space)



• Basis vector: $\vec{a} = 3\vec{i} + 2\vec{j} - 5\vec{k}$

• Column Vectors: $\vec{a} = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}$

• Magnitude of a vector: $|\vec{a}| = \sqrt{3^2 + 2^2 + (-5)^2} = \sqrt{38}$

• Unit vector: $\frac{\vec{a}}{|\vec{a}|} = \frac{3\vec{i} + 2\vec{j} - 5\vec{k}}{\sqrt{3^2 + 2^2 + (-5)^2}} = \frac{3\vec{i} + 2\vec{j} - 5\vec{k}}{\sqrt{38}}$ OR $\frac{3}{\sqrt{38}}\vec{i} + \frac{2}{\sqrt{38}}\vec{j} - \frac{5}{\sqrt{38}}\vec{k}$

Example 1) Find the unit vector of $\vec{v} = 5\vec{i} - 2\vec{j} + \vec{k}$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{5\vec{i} - 2\vec{j} + \vec{k}}{\sqrt{5^2 + (-2)^2 + 1^2}} = \frac{5\vec{i} - 2\vec{j} + \vec{k}}{\sqrt{30}} \quad \text{OR} \quad \frac{5}{\sqrt{30}}\vec{i} - \frac{2}{\sqrt{30}}\vec{j} + \frac{1}{\sqrt{30}}\vec{k}$$

Example 2) Given P(-3, 1, 2) and Q(1, -1, 3)

a) Find \vec{QP}

$$\vec{QP} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$$

OR $4\vec{i} - 2\vec{j} + \vec{k}$

b) Find $\left| \frac{1}{2}\vec{QP} \right| = \left| \frac{1}{2}(4\vec{i} - 2\vec{j} + \vec{k}) \right|$

$$= |2\vec{i} - \vec{j} + \frac{1}{2}\vec{k}|$$

$$= \sqrt{4 + 1 + \frac{1}{4}} = \sqrt{\frac{21}{4}}$$

1. Given $\vec{a} = \vec{i} - 4\vec{j}$ and $\vec{b} = 3\vec{i} + 2\vec{j}$

a) Find $\vec{a} + 2\vec{b}$

$$= (\vec{i} - 4\vec{j}) + 2(3\vec{i} + 2\vec{j}) = (7\vec{i} + 0\vec{j})$$

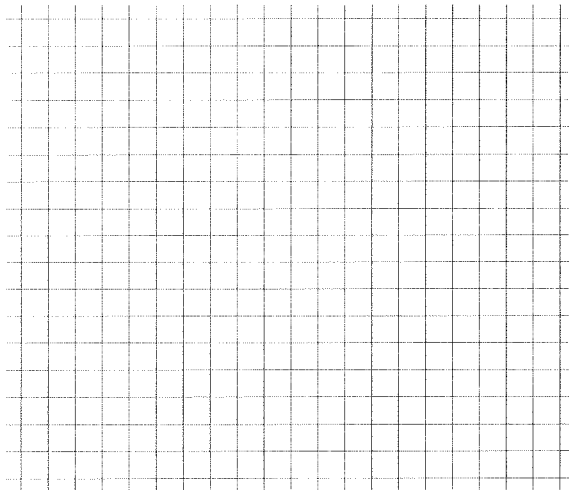
b) Find $\vec{b} - \vec{a}$

$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2\vec{i} + 6\vec{j}$$

c) Illustrate the resultant vectors of $\vec{a} + 2\vec{b}$ and $\vec{b} - 2\vec{a}$ on the grid paper below.

1) $\vec{a} + 2\vec{b}$

2) $\vec{b} - 2\vec{a}$



2. Given $\vec{a} = (-5\vec{j} + w\vec{k})$ is perpendicular to $\vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{a} + 2\vec{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, what is the value of w ?

$$\vec{a} + 2\vec{b} = \begin{pmatrix} 1 \\ -5 \\ w \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ w-6 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow w-6=2$$

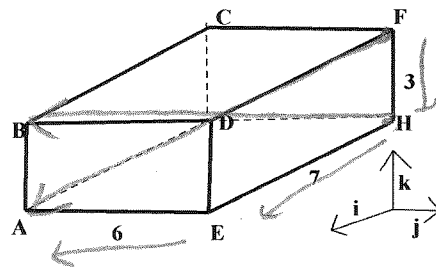
$$\boxed{w=8}$$

3. Consider the prism shown

a. Find the vectors \vec{FA} and \vec{HB} .

$$\vec{FA} = 7\vec{i} - 6\vec{j} - 3\vec{k}$$

$$\vec{HB} = -7\vec{i} - 6\vec{j} + 3\vec{k}$$



b. Find $|\vec{HB}|$

$$= |2(-7\vec{i} - 6\vec{j} + 3\vec{k})| = |-14\vec{i} - 12\vec{j} + 6\vec{k}| = \sqrt{14^2 + 12^2 + 6^2}$$

$$= \sqrt{376}$$

c. Find the vector in direction of \vec{FA} with magnitude of 3 units.

$$= \frac{(7\vec{i} - 6\vec{j} - 3\vec{k})}{\sqrt{376}} = \frac{7}{\sqrt{376}}\vec{i} - \frac{6}{\sqrt{376}}\vec{j} - \frac{3}{\sqrt{376}}\vec{k}$$