Quiz Questions:

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① Find the angle between $a = \begin{pmatrix}3\\4\\7\end{pmatrix}$ and $b = \begin{pmatrix}-3\\8\\2\end{pmatrix}$ in arccos. (3)

$$\cos\theta = \frac{a \cdot b}{|a|\,|b|}$$

$$= \frac{3(-3)+4(8)+7(2)}{\sqrt{3^2+4^2+7^2}\,\sqrt{-3^2+8^2+2^2}} = \frac{-9+32+14}{\sqrt{9+16+49}\,\sqrt{9+64+4}}$$

$$= \frac{37}{\sqrt{74}\,\sqrt{77}} \rightarrow \boxed{\theta = \arccos\left(\frac{37}{\sqrt{74}\,\sqrt{77}}\right)}$$

② Find 'k' if $\vec{a} = \begin{pmatrix}1\\k^2\\k+2\end{pmatrix}$ is perpendicular to $\vec{b} = \begin{pmatrix}2\\1\\-5\end{pmatrix}$ (2)

$$\vec{a} \cdot \vec{b} = \emptyset$$

$$\emptyset = 1(2) + k^2(1) + (k+2)(-5)$$

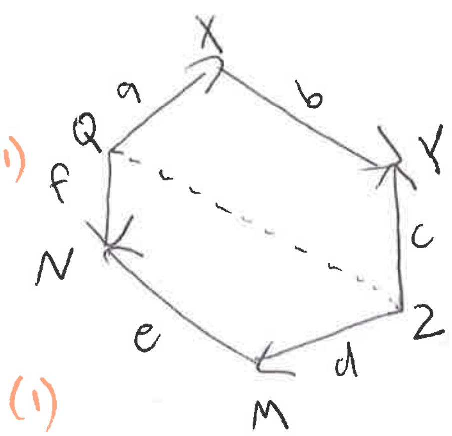
$$\emptyset = 2 + k^2 - 5k - 10$$

$$k^2 - 5k - 8 = \emptyset$$

$$k = \frac{5 \pm \sqrt{25+32}}{2}$$

$$\boxed{k = \frac{5 \pm \sqrt{57}}{2}}$$

③ Write



$\overrightarrow{QZ}$ in terms of a, b, and c (1)

$$\overrightarrow{QZ} = \vec{a} + \vec{b} - \vec{c}$$

$\overrightarrow{QZ}$ in terms of f, e, and d (1)

$$\overrightarrow{QZ} = \vec{f} - \vec{e} - \vec{d}$$

1. Find a and b if R(2,3,0), M(5,-2,6) and S(a,4,b) are collinear.

$\Rightarrow \overrightarrow{RM} = t \cdot \overrightarrow{MS}$

+1 (setting up equations)

$\Rightarrow \overrightarrow{RM} = \begin{pmatrix}5\\-2\\6\end{pmatrix} - \begin{pmatrix}2\\3\\0\end{pmatrix} = \begin{pmatrix}3\\-5\\6\end{pmatrix}$

$\Rightarrow \overrightarrow{MS} = \begin{pmatrix}a\\4\\b\end{pmatrix} - \begin{pmatrix}5\\-2\\6\end{pmatrix} = \begin{pmatrix}a-5\\4+2\\b-6\end{pmatrix}$

1. $3 = t \cdot (a-5)$
2. $-5 = 6t$
3. $6 = t(b-6)$

$\Rightarrow t = -5/6$

+1 (correct answers)

$\Rightarrow \boxed{a = -\frac{7}{5}, \; b = -\frac{6}{5}}$

2. $a = \begin{pmatrix}5\\x\\3\end{pmatrix}$, $b = \begin{pmatrix}6\\-x\\2\end{pmatrix}$; find x so that the angle between a & b is 90°

$\Rightarrow \cos\theta = \frac{a \cdot b}{|a| \cdot |b|}$ * if $a \cdot b = 0$, then $\theta = 90°$

+1 (setting equation)

$\Rightarrow \frac{30 - x^2 + 6}{(\sqrt{34+x^2})(\sqrt{40+x^2})} = 36 - x^2 = 0$

$x^2 = 36$ +1 (getting correct answer)

$\boxed{x = \pm 6}$

3. Let $p = ai + bj + \sqrt{39}k$. If $|\vec{p}| = 8$, find a and b such that both a and b are integers, and $a > b$

$\Rightarrow |\vec{p}| = \sqrt{a^2 + b^2 + (\sqrt{39})^2} = 8$ +1 (setting correct equation)

$= a^2 + b^2 + 39 = 64$

$= a^2 + b^2 = 25$

$= a = 3, b = 4$ or $\boxed{a = 4, b = 3}$ +1 (correct answer)

Catherine Li, Vidya V., Sasha K., Vishal C.

1. If vectors $3i + 5j + 4k$ is parallel to $6i + rj + sk$ solve for variables r and s

Answer: $\begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} = k \begin{pmatrix} -6 \\ r \\ s \end{pmatrix}$

$3 = -6k$

$k = -1/2$

$5 = -\frac{r}{2}$ $\boxed{r = -10}$ $\quad$ $4 = -\frac{s}{2}$ $\boxed{s = -8}$

2. suppose $\vec{a} = \begin{pmatrix} 8 \\ 3 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 7 \\ -9 \\ 1 \end{pmatrix}$

if $2a - 6x = b$, find x

answer: $2a - 6x = b$

$-6x = b - 2a$

$x = -1/6\,(b - 2a)$

$x = -1/6 \begin{pmatrix} 7 \\ -9 \\ 1 \end{pmatrix} \begin{pmatrix} -16 \\ -6 \\ -4 \end{pmatrix}$

$x = -1/6 \begin{pmatrix} -112 \\ 54 \\ -4 \end{pmatrix}$

$x = \begin{pmatrix} 112/6 \\ -9 \\ 2/3 \end{pmatrix}$

3. FIND m and c if a and b are colinear

where $a = (-2, 3, m)$ and $b = (3c, 2, 7)$

$a = tb$

$\to \begin{pmatrix} -2 \\ 3 \\ m \end{pmatrix} = t \begin{pmatrix} 3c \\ 2 \\ 7 \end{pmatrix} \to$

$-2 = 3ct$

$3 = 2t \to \underline{t = \frac{3}{2}}$

$m = 7t$

$-2 = 3c\left(\frac{3}{2}\right)$ $\quad$ $m = 7\left(\frac{3}{2}\right)$

$-2 = \frac{9}{2}c$ $\quad$ $\boxed{m = \frac{21}{2}}$

$\boxed{c = \frac{-4}{9}}$

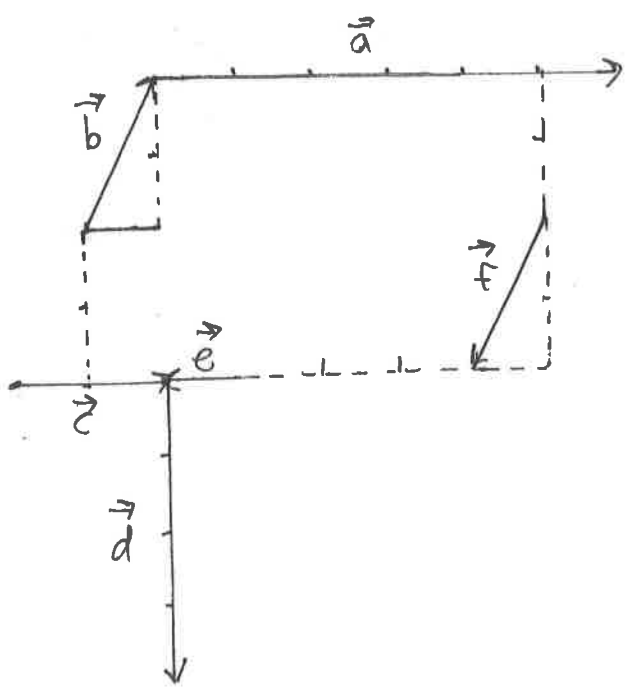
Jason K, Matteo S, Victoria

1) Find $a \cdot b$ if

$|a| = 16$, $|b| = 7$, $\theta = 60°$

$$a \cdot b = |a| \cdot |b| \cdot \cos\theta$$
$$= 16 \cdot 7 \cdot \cos 60°$$
$$= \frac{112}{2}$$
$$= \boxed{56}$$

2) Express vector a in terms of other vectors



$$\boxed{\vec{a} = -2\vec{b} + \vec{c} - 4\vec{e} - 2\vec{f}}$$

graph is drawn to scale.

3) Consider the diagram below

Find the values a & b if X(4, a, 9), Y(2, -1, 3), & Z(b, 5, 11)

$$\overrightarrow{XY} = t \cdot \overrightarrow{YZ}$$

$$\overrightarrow{XY} = \begin{pmatrix} 2-4 \\ -1-a \\ 3-9 \end{pmatrix} = \begin{pmatrix} -2 \\ -1-a \\ -6 \end{pmatrix} \qquad \overrightarrow{YZ} = \begin{pmatrix} b-2 \\ 5+1 \\ 11-3 \end{pmatrix} = \begin{pmatrix} b-2 \\ 6 \\ 8 \end{pmatrix}$$

$-2 = t\cdot(b-2)$

$\frac{8}{3} = b-2$

$\boxed{b = \frac{17}{3}}$

$-1-a = t \cdot 6$

$-1-a = \frac{-9}{2}$

$\boxed{a = \frac{11}{2}}$

$-6 = t \cdot 8$

$t = \frac{-3}{4}$

Alexis S., Rachel W., Steven G., Junior P.

1. Find the velocity vector of a car moving in the direction $-12i+3j$ with the speed 60 mph.

1. convert $-12i+3j$ into unit vector:

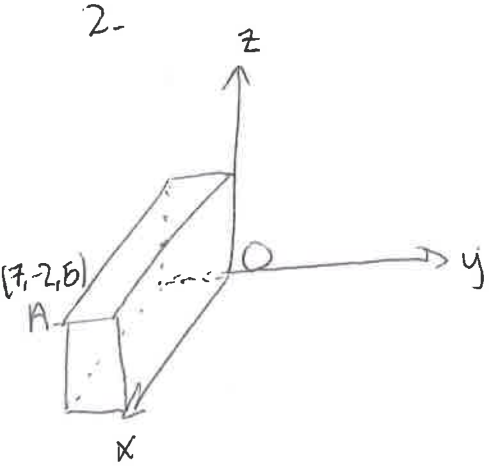
$$|\vec{a}| = \sqrt{12^2+3^2} \rightarrow \sqrt{153}$$

$$\frac{-12i+3j}{\sqrt{153}}$$

2. multiply unit vector times speed

$$\boxed{60 \cdot \left(\frac{-12i+3j}{\sqrt{153}}\right)}$$

2.



a. Find $\vec{OA}$ in unit vector form:

$$\vec{OA} = 7i - 2j + 5k$$

b. Find the angle between $\vec{OA}$ and the

i. x-axis:

$$\begin{pmatrix}7\\-2\\5\end{pmatrix} \cdot \begin{pmatrix}1\\0\\0\end{pmatrix} = 7 \quad \theta = \cos^{-1}\left(\frac{7}{\sqrt{78}}\right)$$

$$\boxed{\theta = 37.6^\circ}$$

ii. y-axis:

$$\begin{pmatrix}7\\-2\\5\end{pmatrix} \cdot \begin{pmatrix}0\\1\\0\end{pmatrix} = -2 \quad \theta = \cos^{-1}\left(\frac{-2}{\sqrt{78}}\right)$$

$$\boxed{\theta = 103.1^\circ}$$

iii. z-axis:

$$\begin{pmatrix}7\\-2\\5\end{pmatrix} \cdot \begin{pmatrix}0\\0\\1\end{pmatrix} = 5 \quad \theta = \cos^{-1}\left(\frac{5}{\sqrt{78}}\right)$$

$$\boxed{\theta = 55.5^\circ}$$

3. Using the vecters $\vec{a} = 3i + 4j - 2K$ and $\vec{b} = -2i - j + K$

a) Find the angle between the vecters $\vec{a}$ and $\vec{b}$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

$$\vec{a} \cdot \vec{b} = 3(-2) + 4(-1) - 2(1) = -6 - 4 - 2 = -12$$

$$\cos\theta = \frac{-12}{\sqrt{29}\sqrt{6}}$$

$$|a| = \sqrt{3^2 + 4^2 - 2^2} = \sqrt{29}$$

$$|b| = \sqrt{-2^2 - 1^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\boxed{\arccos\left(\frac{-12}{\sqrt{29}\sqrt{6}}\right) = \theta}$$ or $$\boxed{\theta = 155°}$$

b) using the answer, is the angle obtuse, acute, or perpendicular?

$$\boxed{\text{obtuse}}$$

1. $\vec{a} = 2i + 5k - 7j$ $\quad \vec{b} = 3j + 10j$ Final

write $3\vec{a} + 2\vec{b}$ in compound form

$\vec{a} = \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$ $\quad \vec{b} = \begin{pmatrix} 3 \\ 10 \\ 0 \end{pmatrix}$

$3\vec{a} + 2\vec{b}$

$3\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 10 \\ 0 \end{pmatrix}$

$\begin{matrix} 6 \\ -21 \\ 15 \end{matrix} + \begin{matrix} 6 \\ 20 \\ 0 \end{matrix}$

$\begin{matrix} 12 \\ -1 \\ 15 \end{matrix}$

2. Find a and b if K(2, -3, 1), L(5, -2, 6), and M(a, 2, b) are collinear.

$\vec{KL} = \begin{pmatrix} 5-2 \\ -2+3 \\ 6-1 \end{pmatrix} \to \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$

$\vec{LM} = \begin{pmatrix} a-5 \\ 2+2 \\ b-6 \end{pmatrix} \to \begin{pmatrix} a-5 \\ 4 \\ b-6 \end{pmatrix}$

$3 = t(a-5)$

$1 = t4 \to t = 1/4$

$5 = t(b-6)$

$3 = 1/4(a-5)$

$12 = a-5$

$\boxed{17 = a}$

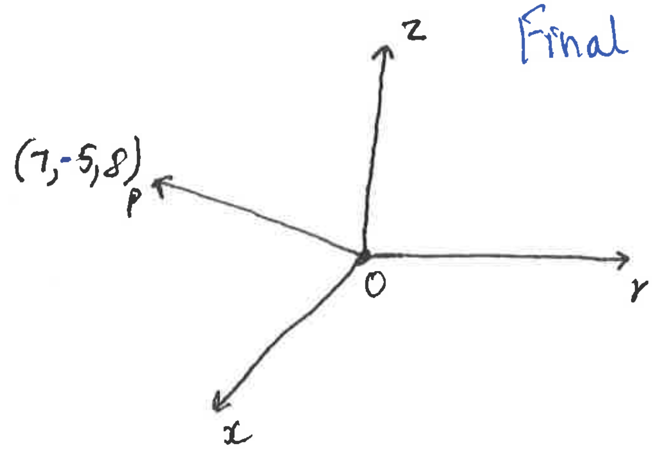
$5 = 1/4(b-6)$

$20 = b-6$

$\boxed{b = 26}$

Final 3rd question Shreyas Madhwaraju

3.)



a) Write $\overrightarrow{OP}$ in terms of the base unit vectors

ans) $\overrightarrow{OP} = \begin{pmatrix} 7 \\ -5 \\ 8 \end{pmatrix} \Rightarrow 7i - 5j + 8k$

b) find the angle $\overrightarrow{OP}$ forms with:

(i) x-axis

(ii) y-axis

(iii) z-axis

ans) (i) $\cos\theta = \dfrac{\overrightarrow{OP} \cdot \overrightarrow{Ox}}{|\overrightarrow{OP}|\,|\overrightarrow{Ox}|}$

$\cos\theta = \dfrac{7}{\sqrt{7^2+5^2+8^2} \times \sqrt{1^2}}$

$\cos\theta = \dfrac{7}{\sqrt{130}}$

$\theta = \cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right)$

$= \boxed{52.1°}$

(ii) $\cos\theta = \dfrac{-5}{\sqrt{130}}$

$\theta = \cos^{-1}\left(\dfrac{-5}{\sqrt{130}}\right)$

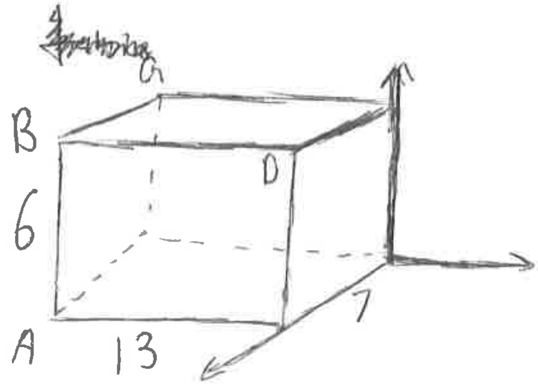
$= \boxed{116°}$

(iii) $\cos\theta = \dfrac{8}{\sqrt{130}}$

$\theta = \cos^{-1}\left(\dfrac{8}{\sqrt{130}}\right)$

$\boxed{\theta = 45.4°}$

1) consider the prism shown:



Find unit vector of $\vec{OB}$

$\vec{OB} = \begin{pmatrix} 7 \\ -13 \\ 6 \end{pmatrix}$

$\vec{OB} = 7i - 13j + 6k$

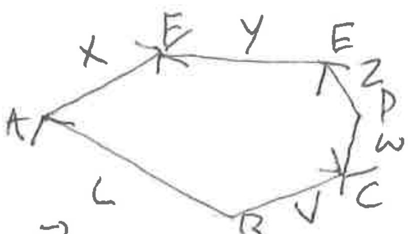
$$\frac{7i - 13j + 6k}{\sqrt{49+169+36}} = \boxed{\frac{7}{\sqrt{254}}i - \frac{13}{\sqrt{254}}j + \frac{6}{\sqrt{254}}k}$$

2) Find the angle between $a = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ -1 \\ 3 \end{pmatrix}$

$\cos\theta = \frac{a \cdot b}{|a||b|}$ $\qquad \cos\theta = \frac{-10+3+6}{\sqrt{25+9+4}\sqrt{4+1+9}}$

$\cos^{-1}\left(\frac{-1}{\sqrt{38}\sqrt{14}}\right) = \boxed{92.5°}$

3)



a) write $\vec{BE}$ in terms of L, x, & y

$\boxed{L + x \quad y}$

b) write $\vec{BE}$ in terms of v, w, & z

$\boxed{v - w + z}$

Conner, Amanda, Riddhivity, Declan

Craig Katsube, Noah Yamashita, Alexei Zhang, Diana Sambotin

1. Find the angle between $\vec{x} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{y} = \begin{pmatrix} 0 \\ 9 \\ 4 \end{pmatrix}$.

$$x \cdot y = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 9 \\ 4 \end{pmatrix}$$
$$= (3)(0) + (2)(9) + (3)(4)$$
$$= 0 + 18 + 12$$
$$= 30$$
$$|x| = \sqrt{9+4+9} = \sqrt{22}$$
$$|y| = \sqrt{0+81+16} = \sqrt{97}$$

$$\cos\theta = \frac{x \cdot y}{\sqrt{x}\sqrt{y}}$$
$$\cos\theta = \frac{30}{\sqrt{22}\sqrt{97}}$$
$$\theta = \cos^{-1}\left(\frac{30}{\sqrt{22}\sqrt{97}}\right)$$
$$\boxed{\theta \approx 49.5^\circ}$$

2. Find $\vec{r}$ and $\vec{s}$ so that $\vec{a}$ and $\vec{b}$ are parallel if $\vec{a} = \begin{pmatrix} 5 \\ 3 \\ r \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} s \\ 1 \\ -5 \end{pmatrix}$.

$$\begin{pmatrix} 5 \\ 3 \\ r \end{pmatrix} = t \begin{pmatrix} s \\ 1 \\ -5 \end{pmatrix} \Rightarrow \begin{array}{l} 5 = t \cdot s \\ 3 = t \cdot 1 \rightarrow t = 3 \\ r = t \cdot (-5) \end{array}$$

$$\boxed{\vec{s} = 5/3}$$
$$\boxed{\vec{r} = -15}$$

3. Given points $A(2,5,1)$ $B(1,6,0)$ $C(0,10,4)$, Find $\vec{AB} \cdot \vec{AC}$

$$\vec{AB} = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$
$$\vec{AC} = \begin{pmatrix} 0 \\ 10 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$$
$$\vec{AB} \cdot \vec{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} = (-2)(-1) + (5)(1) + (3)(-1)$$
$$= 2 + 5 - 3$$
$$\boxed{\vec{AB} \cdot \vec{AC} = -4}$$