4th period

Final Review Notes

2016 ~ 2017

Chapter 17: Limits:

Definition: If f(x) can be made as close as we like to some real number A by making x sufficiently close to (but not equal to) A, then we say that f(x) has a limit of A as x approaches a.

$\lim_{x \to a} f(x) = A$ Note: Removal discontinuity = hole & there is still a limit there → AKA: Hole = ¬ <u>not</u> continuous

F is continuous at x = a if...

1. f(a) is defined
2. $\lim f(x)$ exists
3. $f(a) = \lim_{x \to a} f(x)$

Limits At Infinity:

$\lim_{x \to \infty} \frac{1}{x} = 0$

This is b/c as x → 0 & as x gets large, $\frac{1}{x}$ gets small

Rules:

$\lim_{x \to a} c = c$

$\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

$\lim_{x \to a} [f(x) g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} f(x) \div \lim_{x \to a} g(x)$

Trigonometric Limits:

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ Note: x <u>has</u> to be approaching 0

Note: $\lim_{x \to 0} \frac{\sin 4x}{7x} \rightarrow \frac{4}{7}$ is answer

* x in <u>radians</u> *

Chapter 17: Definition of Derivatives:

Derivative: <u>Instantaneous</u> rate of change → AKA: Slope of tangent lines

Slope of Secant Line:

$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

↳ Difference quotient

Slope Of Tangent Line:

$$\lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x)$$

↳ First Principle

↳ Def. of a derivative using a limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = \frac{dy}{dx}$$

Note: This is of the point (x+h, f(x+h))

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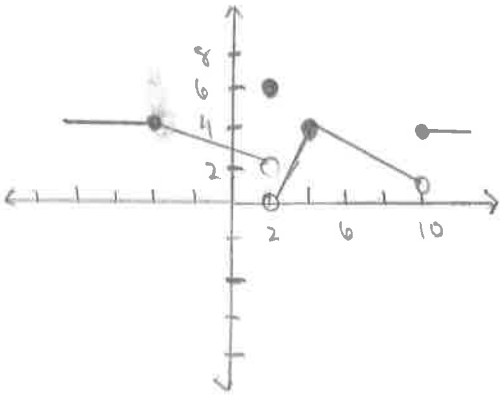
IB HL 1 Period 4 S1 Review

Limit and Definition of Derivative Examples

ex 1.



a) $\lim_{x\to 2^-} f(x) = 2$

b) $\lim_{x\to 2} f(x) = \text{DNE}$

c) $\lim_{x\to 10^-} f(x) = 1$

d) $f(2) = 6$

ex 2. $\lim_{x\to 2} \frac{x-2}{x^3-8}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)}$

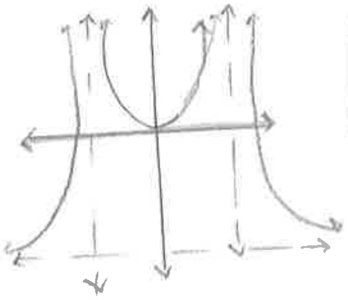
$\lim_{x\to 2} \frac{1}{x^2+2x+4}$

$\lim_{x\to 2} \frac{1}{(2)^2+2(2)+4}$

$= \boxed{\frac{1}{12}}$

ex 3. $\lim_{x\to\infty} \frac{5x-7x^2}{x^2-16}$ HA: $y = -7$

$\lim_{x\to\infty} \frac{x(5-7x)}{(x-4)(x+4)}$ VA : $x = \pm 4$



$\boxed{\lim_{x\to\infty} = -7}$

ex 5. Find slope of tangent line at $x=3$

to : $f(x) = -2x^2+5x$

using $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0} \frac{-2(x+h)^2+5(x+h)+2x^2-5x}{h}$

$\lim_{h\to 0} \frac{-2x^2-4xh-2h^2+5x+5h+2x^2-5x}{h}$

$\lim_{h\to 0} \frac{-4xh-2h^2+5h}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(-4x-2h+5)}{\cancel{h}}$

$\lim_{h\to 0} -4x-2h+5$

$\lim_{h\to 0} -4(3)-2(0)+5 = \boxed{-7}$

ex 4
$$f(x) = \begin{cases} Ax+6 & \text{if } x<2 \\ -7 & \text{if } x=2 \\ x^2+Ax+B & \text{if } x>2 \end{cases}$$

$\lim_{x\to 2^-} Ax+6$ $\lim_{x\to 2} = -7$ $\lim_{x\to 2^+} x^2+Ax+B$

$\lim_{x\to 2^-} Ax+6 = \lim_{x\to 2} -7$

$2A+6 = -7$

$\boxed{A = -13/2}$

$\lim_{x\to 2^+} (2)^2 - 13/2\,(2) + B = \lim_{x\to 2} -7$

$4-13+B = -7$

$\boxed{B = 2}$

Trig. Log. Differentiation

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3. $y=(x^2+7)^x$

$\ln y=\ln(x^2+7)^x$ – take natural log of both

$\ln y=x\ln(x^2+7)$ – move exponent to front

$\frac{dy}{dx}\left(\frac{1}{y}\right)=x\left(\frac{2x}{x^2+7}\right)+1(\ln(x^2+7))$ – Take Derivative

$\frac{dy}{dx}\left(\frac{1}{y}\right)=\frac{2x^2}{x^2+7}+\ln(x^2+7)$ – Simplify

$\frac{dy}{dx}\left(\frac{1}{y}\right)\cdot y=y\left(\frac{2x^2}{x^2+7}+\ln(x^2+7)\right)$ – multiply by "y"

$\boxed{\frac{dy}{dx}=(x^2+7)^x\left(\frac{2x^2}{x^2+7}+\ln(x^2+7)\right)}$ – substitute for y

To differentiate an equation with a variable in the exponent, take the natural log of both sides, and then take the derivative of both sides, substitute and simplify.

Arc - Trig Differentian

If $y = \arcsin x$, then $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

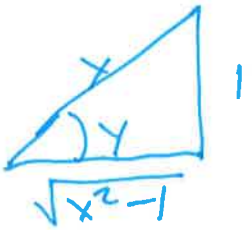
$x \in]-1, 1[$

If $y = \arccos x$, then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$, $x \in]-1, 1[$.

If $y = \arctan x$, then $\frac{dy}{dx} = \frac{1}{1+x^2}$, $x \in R$

Proof for $\csc^{-1} x = y$:

$\csc y = x$



$1 = \csc y \cot y \frac{dy}{dx}$

$\frac{1}{-\csc y \cot y} = \frac{dy}{dx}$

$\frac{1}{-x\sqrt{x^2-1}} \rightarrow \frac{-1}{|x|\sqrt{x^2-1}}$

pg 556 #3a

$y = \arctan(2x)$

$\frac{dy}{dx} = 2 \times \frac{1}{1+(2x)^2} = \frac{2}{1+4x^2}$

Equations of Tangent and Normal

Tangent

Derivative = slope(m) Point-slope form: $y-y_1=m(x-x_1)$

Ex: Find the equation of the tangent to $y=3x^2+6x+4$ at $x=\underset{x_1}{2}$

$\frac{dy}{dx}=6x+6$ $\frac{dy}{dx}\Big|_{x=2}=6(2)+6=\underset{m}{18}$ $y=3(2)^2+6(2)+4=\underset{y_1}{28}$

$y-28=18(x-2)$ $y-28=18x-36$ $\boxed{y=18x-8}$

Normal

Perpendicular to the tangent $\left(\frac{-1}{m}\right)$

Ex: Find the equation of the normal to $y=3x^2+6x+4$ at $x=\underset{x_1}{2}$

$\frac{dy}{dx}=6x+6$ $\frac{dy}{dx}\Big|_{x=2}=6(2)+6=18$ $y=3(2)^2+6(2)+4=\underset{y_1}{28}$

$m=-\frac{1}{18}$ $y-28=-\frac{1}{18}(x-2)$ $y-28=-\frac{1}{18}x+\frac{1}{9}$

$\boxed{y=-\frac{1}{18}x+\frac{253}{9}}$

CURVE ANALYSIS

Stationary inflection - inflection pt. and critical pt.

First Derivative Test

- Critical Points: where $f'(c)=0$ (aka Stationary Point)
  - if it goes from increasing to decreasing it is a local maximum ↗↘
  - if it goes from decreasing to increasing it is a local min ↘↗

Second Derivative Test

- if $f''(c)<0$, it is concave down ↶
- if $f''(c)>0$, it is concave up ↷
- if $f''(c)=0$, it is an inflection point

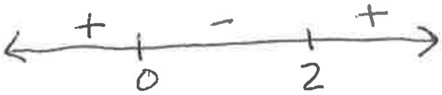
Example: Sketch the graph of $f(x)=x^4-4x^3+2$

a. Make sign diagram of 1st + 2nd derivative.

$f'(x)=4x^3-12x^2$

critical points: $x=0$ $x=3$

$f''(x)=12x^2-24x$



inflection points: $x=2$ $x=0$

b. Find intervals where $f(x)$ is

Increasing:	Decreasing:	Concave Up:	Concave Down:
$(3,\infty)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(2,\infty)$	$(0,2)$

c. Give coordinates where $f(x)$ has a

Local max	Local min	Stationary Inflection
none	$(3,-25)$	$(0,2)$

Non-Stationary Inflection
$(2,-14)$

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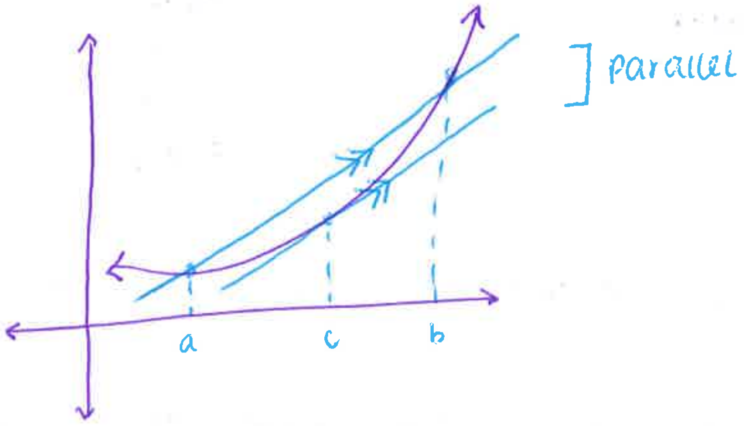
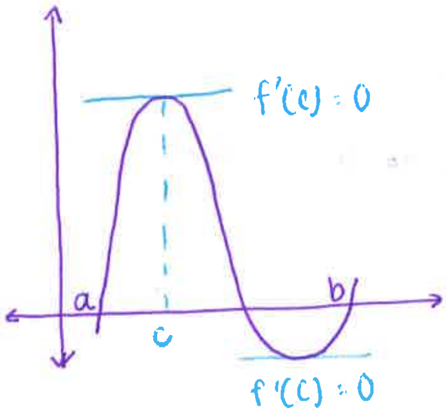
Group 5: MVT, Rolle's Theorem, Kinematics

Mean Value Theorem: $f'(c) = \frac{f(b) - f(a)}{b - a}$



Rolle's Theorem: If $f(a) = f(b)$, then there exists a value c such that $f'(c) = 0$



This theorem can help identify if there is a local max/min in a graph.

Kinematics:

$s(t) =$ position
$v(t) = s'(t)$
$a(t) = s''(t)$

ex: position of a particle

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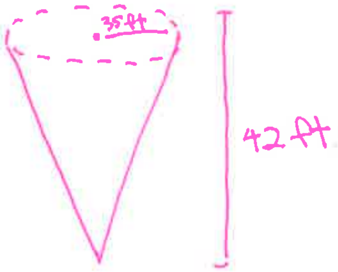
Related Rates

Example:

An underground conical tank, standing on its vertex, is being filled w/ water at the rate of 56 ft³/min. The tank has a height of 42 ft and a radius of 35 ft. How fast is the water top surface area changing when the water is 16 ft deep?



$= \frac{h}{r} = \frac{42}{35}$

$h = \frac{6}{5}r$

$16 = \frac{6}{5}r$

$r = \frac{40}{3}$

$\frac{dV}{dt} = 56\ ft^3/min$

$r = \frac{5}{6}h$

$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = \frac{1}{3}\pi \cdot r^2 \cdot \frac{6}{5}r$

$\frac{dV}{dt} = \frac{1}{3}\pi \cdot \frac{6}{5}r^3$

$\frac{dV}{dt} = \frac{1}{3}\pi \cdot 3 \cdot \frac{6}{5} \cdot r^2 \cdot \frac{dr}{dt}$

$56 = \frac{1}{3}\pi \cdot 3 \cdot \frac{6}{5} \cdot r^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = 0.0836$

$A = \pi r^2$

$\frac{dA}{dt} = \pi \cdot 2 \cdot \frac{40}{3} \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = \pi \cdot 2 \cdot \frac{40}{3} \cdot 0.0836$

$\frac{dA}{dt} = \boxed{7\ ft^2/min}$

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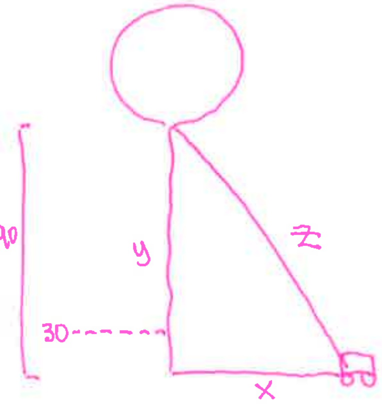
OPTIMIZATION

Steps:

1. Draw a diagram and state the given information by defining the variables
2. Set up proper mathematical equations using variables
3. Diffrentiate the equations w/ respect to time. Or find the optimum value for the appropriate variable.

Example:

A hot air balloon rises vertically at a rate of 60 ft/s. A car travelling at 80 ft/s drives directly under the balloon. The back of the car is directly below the balloon when the balloon is 30 ft above the ground. How fast is the distance between the balloon and car changing exactly one second after the back of the car is directly below the balloon?



$z^2 = x^2 + y^2$

$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$

$2(120) \cdot \frac{dz}{dt} = 2(80)(80) + 2(90)(60)$

$240 \cdot \frac{dz}{dt} = 23600$

$\frac{dz}{dt} = \boxed{98.0 \text{ ft/s}}$

$t = 1\text{s}$

$\frac{dy}{dt} = 60 \text{ ft/s}$

$\frac{dx}{dt} = 80 \text{ ft/s}$

$y = 60 \cdot 1 + 30 = 90 \text{ ft}$

$x = 80 \cdot 1 = 80 \text{ ft}$

$z^2 = x^2 + y^2$

$z^2 = 80^2 + 90^2$

$z = 120 \text{ ft}$

$\frac{dz}{dt} = ?$

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Optimization

A mathematical technique for finding a maximum or minimum value of a function of several variables subject to a set of constraints, such as linear programming or systems analysis.

In human terms: using derivates to get the most effecient/greatest/smallest values, usually for real-world examples.

Related Rate:

Finding the rate at which a quantity changes by relating the quantity to other quantities whose rates of change are known.

In human terms: The rate at which things travel relative to other quantities given

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Chapter 14: Vectors

Vectors - magnitude and direction "See Vector from Despicable Me"

Scalars - only magnitude

Parallelism - $\vec{A} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$ $\vec{B} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$ $\vec{A}$ and $\vec{B}$ are parallel

Dot Product - $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

Cross Product - $\vec{a} \times \vec{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ -(a_1 b_3 - a_3 b_1) \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

Applications - physics, currents, displacement

Collinear - Point a = (1 , 7 , 3) Point b = (2 , 7 , 6)
Point c = (3 , w , x)
If collinear, $\vec{ab}$ is parallel to $\vec{bc}$