Quiz Problems

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(1) Find vector $\vec{a}$ if it is in the same direction as vector $\vec{b}$ with magnitude of 5 units if $\vec{b} = \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$.

Solution:

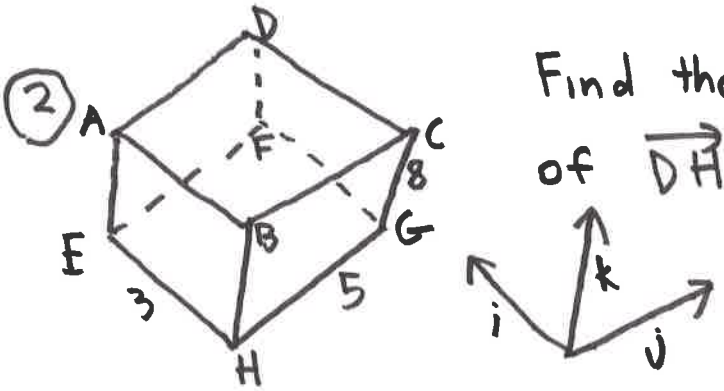
$$|\vec{b}| = \sqrt{1^2 + (-7)^2 + 3^2}$$
$$= \sqrt{1 + 49 + 9}$$
$$= \sqrt{59}$$

$$\frac{\vec{b}}{|\vec{b}|} = \frac{i - 7j + 3k}{\sqrt{59}}$$
$$= \frac{1}{\sqrt{59}} i - \frac{7}{\sqrt{59}} j + \frac{3}{\sqrt{59}} k$$

$$\vec{a} = 5\left(\frac{1}{\sqrt{59}} i - \frac{7}{\sqrt{59}} j + \frac{3}{\sqrt{59}} k\right)$$

$$\boxed{\vec{a} = \frac{5}{\sqrt{59}} i - \frac{35}{\sqrt{59}} j + \frac{15}{\sqrt{59}} k}$$

(2)



Find the vector in the opposite direction of $\overrightarrow{DH}$ with magnitude of 8 units.

Solution:

$$\overrightarrow{DH} = -3i - 5j - 8k$$

$$|\overrightarrow{DH}| = \sqrt{(-3)^2 + (-5)^2 + (-8)^2}$$
$$= \sqrt{9 + 25 + 64}$$
$$= \sqrt{98}$$

$$\frac{\overrightarrow{DH}}{|\overrightarrow{DH}|} = \frac{-3i - 5j - 8k}{\sqrt{98}}$$
$$= -\frac{3}{\sqrt{98}} i - \frac{5}{\sqrt{98}} j - \frac{8}{\sqrt{98}} k$$

$$\text{Vector} = 8\left(-\frac{3}{\sqrt{98}} i - \frac{5}{\sqrt{98}} j - \frac{8}{\sqrt{98}} k\right)$$

$$\boxed{= -\frac{24}{\sqrt{98}} i - \frac{40}{\sqrt{98}} j - \frac{64}{\sqrt{98}} k}$$

(3) Find r and x if vector a is $-3i + 2j - 4k$ and vector b is $-ri + xj - 8k$, a and b are parallel.

Solution:

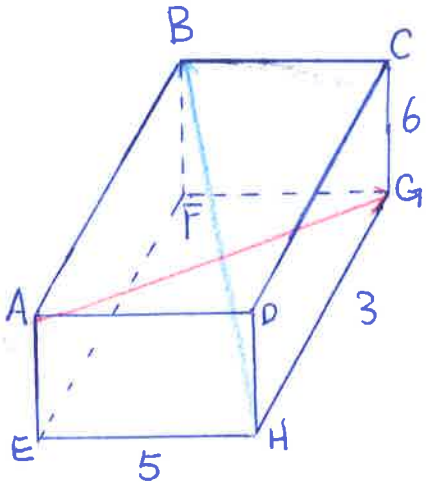
$$\begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix} = t \begin{pmatrix} -r \\ x \\ -8 \end{pmatrix}$$

$-3 = -tr$ $\quad 2 = tx$ $\quad -4 = -8t$

$-3 = -\frac{1}{2} r$ $\quad 2 = \frac{1}{2} x$ $\quad t = \frac{1}{2}$

$\boxed{r = 6}$ $\quad \boxed{x = 4}$

1.



Find $\overrightarrow{AG}$ and $\overrightarrow{HB}$

- $\overrightarrow{AG}: -3i+5j-6k$
- $\overrightarrow{HB}: -3i-5j+6k$

2. Suppose $\vec{a}=\begin{pmatrix}2\\1\\1\end{pmatrix}$ $\vec{b}=\begin{pmatrix}6\\2\\3\end{pmatrix}$

Find X such that $a+2X=b$

$$a-b=-2X$$

$$\begin{pmatrix}2\\1\\1\end{pmatrix}-\begin{pmatrix}6\\2\\3\end{pmatrix}=-2X$$

$$\begin{pmatrix}-4\\-1\\-2\end{pmatrix}=-2X$$

$$X=\begin{pmatrix}2\\\frac{1}{2}\\1\end{pmatrix}$$

3. Suppose $\vec{a}=\begin{pmatrix}3\\-8\\21\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}k^2\\3k\\-10\end{pmatrix}$. Find the constant k given that $\vec{a}$ and $\vec{b}$ are perpendicular

$$\vec{a}\cdot\vec{b}=\begin{pmatrix}3\\-8\\21\end{pmatrix}\cdot\begin{pmatrix}k^2\\3k\\-10\end{pmatrix}=0$$

$$3k^2-24k-210=0$$

$$k^2-8k-70=0$$

$$k=\frac{8\pm\sqrt{8^2-4(1)(-70)}}{2}$$

$$k=\frac{8\pm\sqrt{64+280}}{2}$$

$$k=\frac{8\pm\sqrt{344}}{2}$$

$$k=4\pm\sqrt{86}$$

$$k\approx 13.3 \quad k\approx -5.27$$

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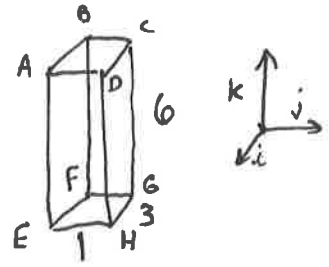
1) 

Find the angle of

a. $\hat{ABH}$

AB: $\begin{pmatrix}-3\\0\\0\end{pmatrix}$ BH: $\begin{pmatrix}-6\\3\\1\end{pmatrix}$

$\begin{pmatrix}-3\\0\\0\end{pmatrix}\cdot\begin{pmatrix}-6\\3\\1\end{pmatrix} = -18+0+0 = -18$

$|AB| = \sqrt{9} = 3$

$|BH| = \sqrt{36+9+1} = \sqrt{46}$

$\cos\theta = \frac{-18}{3\sqrt{46}}$

$\boxed{\theta = 152^\circ}$

2) Find v if: $p = 5i + 7j$ & $q = 3i - vj$ are perpendicular

$\begin{pmatrix}5\\7\end{pmatrix}\cdot\begin{pmatrix}3\\-v\end{pmatrix} = 0$

$15 - 7v = 0$

$7v = 15$

$\boxed{v = \frac{15}{7}}$

3) Find x and y if $x\begin{pmatrix}3\\6\\7\end{pmatrix} + \begin{pmatrix}2\\5\\y\end{pmatrix} = \begin{pmatrix}11\\23\\27\end{pmatrix}$

$3x + 2 = 11$

$\boxed{x = 3}$

$(3)(7) + y = 27$

$\boxed{y = 6}$

Final Draft
Table 1
HL Math 1
P. 4

#1 Points Q, R, and S are collinear. Given the points Q(5, a, 1), R(-3, 7, 2), S(4, 2, b), find a and b.

$\vec{QR}$ $\vec{RS}$

$\begin{bmatrix} -3-5 \\ 7-a \\ 2-1 \end{bmatrix}$ $\begin{bmatrix} 4-{-3} \\ 2-7 \\ b-2 \end{bmatrix}$

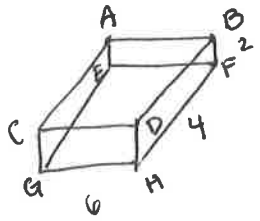
$\begin{pmatrix} -8 \\ 7-a \\ 1 \end{pmatrix}$ $\begin{pmatrix} 7 \\ -5 \\ b-2 \end{pmatrix}$

$1 = \frac{-8}{7}(b-2)$

$\frac{-7}{8} = b-2$ $\quad +2$ $\quad +2$

$b = 9/8$

#2 Find vectors $\vec{AH}$ and $\vec{GB}$ in unit vector form.



$\vec{AH}$: $4i + 6j - 2k$

$\vec{GB}$: $-4i + 6j + 2k$

#3 Given $\vec{a} = i + 2j - k$ and $\vec{b} = 2i - 5j + 3k$, find the angle between $\vec{a}$ and $\vec{b}$.

$$\cos\theta = \frac{(i+2j-k)\cdot(2i-5j+3k)}{\sqrt{1+4+1}\cdot\sqrt{4+25+9}}$$

$$\cos\theta = \frac{2-10-3}{\sqrt{6}\cdot\sqrt{38}}$$

$\cos^{-1}$ $\cos^{-1}$

$$\theta = \cos^{-1}\left(\frac{-11}{\sqrt{6}\sqrt{38}}\right)$$

$$\theta = 137^\circ$$

$$\theta \approx 136.760\,1882$$

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QUIZ QUESTIONS

1. Find the angle between vector $\vec{OP}: \begin{pmatrix} 6 \\ 9 \\ -5 \end{pmatrix}$ and the x-axis

$\cos\theta = \frac{a \cdot b}{|a||b|}$ closest point on x-axis: $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$

$$\cos\theta = \frac{\begin{pmatrix} 6 \\ 9 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}}{\sqrt{36+16+25}\sqrt{36}} = \frac{36}{6\sqrt{77}}$$

$$\theta = \cos^{-1}\left(\frac{36}{6\sqrt{77}}\right) = \boxed{46.9^\circ}$$

2. Find a and b so O (3, 6, 13), P (5, 7, 9), and Q (a, 13, b) are colinear

Colinear $\vec{OP} = \perp \vec{PQ}$

$$\vec{OP} = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 13 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$\vec{PQ} = \begin{pmatrix} a \\ 13 \\ b \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix} = \begin{pmatrix} a-5 \\ 6 \\ b-9 \end{pmatrix}$$

① $2 = t(a-5)$
② $1 = 6t \longrightarrow t = \frac{1}{6}$
③ $9 = t(b-9)$

$2 = \frac{1}{6}a - \frac{5}{6}$ $\quad$ $9 = \frac{1}{6}b - \frac{9}{6}$

$\frac{12}{6} = \frac{1}{6}a - \frac{5}{6}$ $\quad$ $\frac{54}{6} = \frac{1}{6}b - \frac{9}{6}$

$\frac{17}{6} = \frac{1}{6}a$ $\quad$ $\frac{63}{6} = \frac{1}{6}b$

$\boxed{a = 17}$ $\quad$ $\boxed{b = 63}$

3. Find a if $p = 4\vec{i} - 7\vec{j}$ and $q = a\vec{i} + 6\vec{j}$ is perpendicular

$\vec{p} \cdot \vec{q} = 0 \rightarrow 90^\circ$ angle

$4 \cdot a - 42 = 0$

$4a = 42$

$\boxed{a = \frac{21}{2}}$

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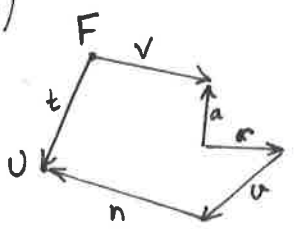
(1.)



$\vec{FU} = v - a + r + u + n$

Find $\vec{FU}$ with vectors other than t:

(2.) object a flies through space in the direction $\begin{pmatrix}3\\1\\1\end{pmatrix}$ object b flies through space in the direction $\begin{pmatrix}4\\-2\\q\end{pmatrix}$, perpendicular to object a's path.

find q.

$\cos 90 = 0 = \frac{\vec{a}\cdot\vec{b}}{|a|\cdot|b|}$

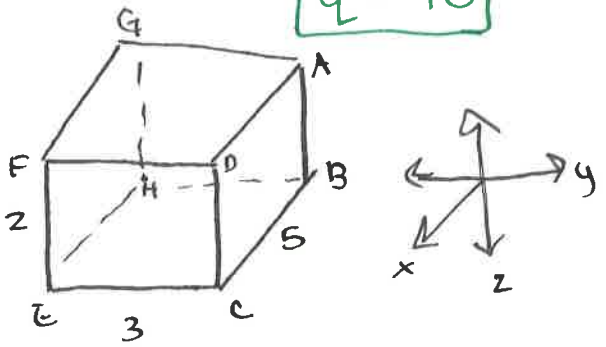
$0 = \vec{a}\cdot\vec{b}$

$0 = \begin{pmatrix}3\\1\\1\end{pmatrix}\cdot\begin{pmatrix}4\\-2\\q\end{pmatrix}$

$0 = 12 + (-2) + q$

$\boxed{q = -10}$

(3.) using the given prism:



a. find the vectors $\vec{FC}$ and $\vec{AE}$

$\vec{FC} = 3y - 2z$

$\vec{AE} = 5x - 3y - 2z$

b. find the angle between $\vec{AE}$ and $\vec{GC}$ ← $5x+3y-2z$

$\cos\theta = \frac{\vec{AE}\cdot\vec{GC}}{|AE|\cdot|GC|}$ $\cos\theta = \frac{5x-3y-2z\ \cdot\ 5x+3y-2z}{\sqrt{38}\quad\sqrt{38}}$ $\sqrt{25+9+4} = \sqrt{38}$

$\cos\theta = \frac{\sqrt{25+9+4}}{38}$

$\cos\theta = \frac{\sqrt{20}}{38}$ $\theta = \arccos\left(\frac{\sqrt{20}}{38}\right)$ $\theta = 83.2°$

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① $\vec{A} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$ $\vec{C} = \begin{pmatrix} x \\ y \\ 4 \end{pmatrix}$

Find x and y if $\vec{C} \parallel \vec{A}$

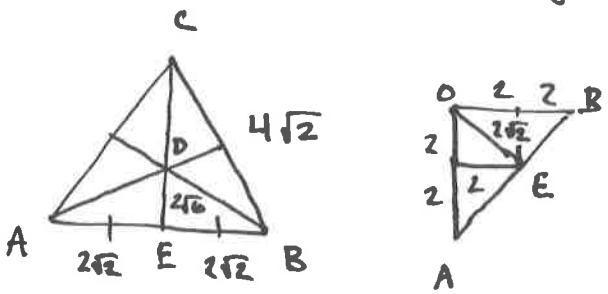
$\begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = t \cdot \begin{pmatrix} x \\ y \\ 4 \end{pmatrix}$

$5 = t \cdot x \quad 3 = t \cdot y \quad 2 = 4t$

$5 = \frac{1}{2} \cdot x \quad 3 = \frac{1}{2} \cdot y \quad t = \frac{1}{2}$

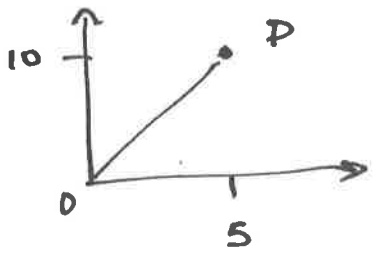
$\boxed{x = 10 \quad y = 6}$

② Vectors $\vec{OA}$, $\vec{OB}$, and $\vec{OC}$ are $\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ respectively. Line segments AB, BC, and AC are drawn to form a triangular pyramid. Let point D be the centroid of triangle ABC. Find the magnitude of $\vec{OD}$.



$\vec{OD} = \begin{pmatrix} 2 \\ 2 \\ 2\sqrt{6} \end{pmatrix}$ $|\vec{OD}| = \sqrt{2^2+2^2+(2\sqrt{6})^2} = \sqrt{32} = \boxed{4\sqrt{2}}$

③ What is $\vec{OP}$ in both unit and component form?

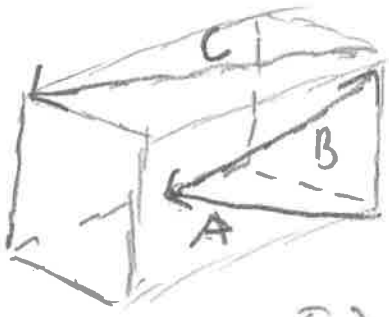


Unit: $5i + 10j$

Component: $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$

work

1.

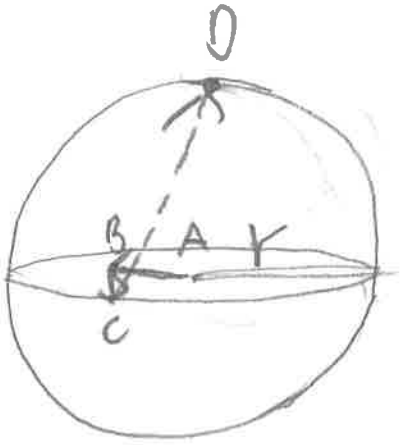


$A = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix}$ $B = \begin{pmatrix} -5 \\ 3 \\ 9 \end{pmatrix}$ $C = \begin{pmatrix} 15 \\ -3 \\ 0 \end{pmatrix}$

What is the dimension of the rectangular prism

15 by 3 by 9

2.



A is the origin

D is on top of A.

$\vec{AB} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$\vec{CD} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$

Find the length of r.

r = 5

3. Missing.