period 5

Final Review Notes

2016 ~ 2017

CHAPTER 17

chapter seventeen

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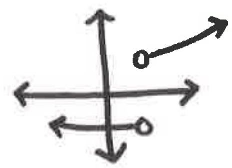
[LIMIT AND DEFINITION OF DERIVATIVE]

LIMITS

DOES NOT EXIST WHEN...

$$\lim_{x=0^-} \neq \lim_{x=0^+}$$



[A VALUE THAT A FUNCTION OR SEQUENCE APPROACHES BUT DOES NOT CONTAIN]

[· Algebraic
· Limits involving ∞
· trig limits]

Types of Limit Problems

EXAMPLE 1: FIND THE LIMIT OF $F(X)=\dfrac{1-\cos^2 x}{6x^2}$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{6x^2}$$

$$\lim_{x\to 0} \frac{\sin^2 x}{6x^2}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{1}{6}$$

$$\boxed{\lim_{x\to 0} \frac{1}{6}}$$

RULES TO REMEMBER:

$$\frac{\sin x}{x} = 1$$

YAY ☺

EXAMPLE 2:

FIND THE CONSTANTS A AND B SO THAT THE GIVEN FUNCTION WILL BE CONTINUOUS FOR ALL X.

$$f(x)\begin{cases} Ax+6 & \text{if } x<2 \\ -7 & \text{if } x=2 \\ x^2+Ax+B & \text{if } x>2 \end{cases}$$

$2A + B = -7$

$2^2 + 2A + B = -7$

$2A + B = -11$

$2A = -13$

$A = \dfrac{-13}{2}$

$2\left(\dfrac{-13}{2}\right) + B = -11$

$-13 + B = -11$

$B = 2$

YOU CAN DO IT! I BELIEVE IN YOU ♡

DERIVATIVE

A FUNCTION WHICH GIVES THE SLOPE OF A CURVE; THE SLOPE OF THE LINE TANGENT TO A FUNCTION

IMPORTANT EQUATIONS

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

EXAMPLE 3:

FIND THE SLOPE OF THE TANGENT LINE TO THE GRAPH OF $\left[f(x) = -2x^2 + 5x \right]$ AT $\left[x = 3 \right]$ USING THE DEFINITION OF DERIVATIVE

$$f'(x) = \lim_{h \to 0} \frac{-2(x+h)^2 + 5(x+h) - (-2x^2+5x)}{h}$$

$$= \lim_{h \to 0} \frac{-2(3+h)^2 + 5(3+h) - (-2(3)^2 + 5(3))}{h}$$

$$= \lim_{h \to 0} \frac{-2(9+6h+h^2) + 15 + 5h + 18 - 15}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-18} - 12h - 2h^2 + \cancel{15} + 5h + \cancel{18} \cancel{-15}}{h}$$

$$= \lim_{h \to 0} \frac{-7h - 2h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-7-2h)}{\cancel{h}}$$

$$= \lim_{h \to 0} -7 - 2h$$

$$= \lim_{h \to 0} -7 - 2(0)$$

$$= \boxed{\lim_{h \to 0} -7}$$

DIFFERENTIABILITY & CONTINUITY

f IS DIFFERETIABLE AT $x = a$ IF $f'(a) = \lim_{h \to 0^+} = f'(a) = \lim_{h \to 0^-}$

AKA IF RIGHT AND LEFT HAND DERIVATIVES ARE EQUAL

CONTINUATY: IF FUNCTION F IS DIFFERENTIABLE AT $X = C$, THEN F IS CONTINUOUS AT $X = C$

EXAMPLE 4:

PROVE THAT

$$f(x) = \begin{cases} \sin x, & x \geq 0 \\ x^2 + 5x, & x < 0 \end{cases}$$

IS CONTINUOUS BUT NOT DIFFERENTIABLE AT $X = 0$

$$\lim_{x \to 0^+} \sin x = \lim_{x \to 0^-} x^2 + 5x$$

$$0 = 0$$

$\therefore$ f(x) is continuous

$$\lim_{x \to 0^+} (\sin x)' = \lim_{x \to 0^-} (x^2 + 5x)'$$

$$\lim_{x \to 0^+} \cos x = \lim_{x \to 0^-} 2x + 5$$

$$1 \neq 5$$

$\therefore$ f(x) is not differentiable

Group 2

Chapter 18 Derivative Rules

Paul D, Sanana S, Visali S, Deepthi S.

$$f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{x \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

Chain Rule:

$$y = g(u) \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Product Rule:

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule:

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain Rule Example:

$$y = \frac{4}{\sqrt{1-2x}}$$

1.) $y = 4u^{-1/2}$ where $u = 1-2x$

2.) $4x\left(-\frac{1}{2}u^{-\frac{3}{2}}\right) \times (-2)$

3.) $4u^{-\frac{3}{2}}$

4.) $4(1-2x)^{-\frac{3}{2}}$

Product Rule Example:

$$y = x^2(x^2-2x)^4$$

$$\frac{dy}{dx} = 2x(x^2-2x)^4 + x^2(4)(x^2-2x)^3(2x-2)$$

$$\frac{dy}{dx} = 2x(x^2-2x)^4 + 4x^2(x^2-2x)^3(2x-2)$$

Quotient Rule

$$y = \frac{1+3x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{3(x^2+1)-(1+3x)(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{3x^2+3-2x-6x^2}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{-3x^2-2x+3}{(x^2+1)^2}$$

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P.5 Group 2

1

Examples

Quotient Rule

$$f(x)=\frac{\cos 5x}{6x^4-x^2} \quad u'=-\sin 5x \quad v'=24x^3-2x$$

$$\frac{(6x^4-x^2)(-\sin 5x\cdot 5)-(24x^3-2x)(\cos 5x)}{(6x^4-x^2)^2}$$

Answer: $$\frac{-5\sin 5x(6x^4-x^2)-(24x^3-2x)(\cos 5x)}{(6x^4-x^2)^2}$$

2 $f(x)=e^{(x^2-2x)}\cot(5x)$

$\cot x=-\csc^2 x$ $\quad$ $e^{f(x)}=e^{f(x)}\cdot f'(x)$

$$[e^{(x^2-2x)}\cdot -\csc^2(5x)\cdot 5+e^{(x^2-2x)}(2x-2)\cdot\cot(5x)]$$

$$-5e^{(x^2-2x)}\csc^2(5x)+e^{(x^2-2x)}\cot(5x)(2x-2)$$

$$\boxed{-5e^{(x^2-2x)}\csc^2(5x)+e^{(x^2-2x)}\cot(5x)(2x-2)}$$

3 $y=\sqrt[3]{x}\,(3x+5x^4)^6$

chain rule and product rule

$$\frac{dy}{dx}=x^{\frac{1}{3}}\cdot(3x+5x^4)^6$$

$$\frac{dy}{dx}=(x^{\frac{1}{3}})(6)(3x+5x^4)^5(3+20x^3)+(3x+5x^4)^6\left(\frac{1}{3}x^{-\frac{2}{3}}\right)$$

$$\boxed{\frac{dy}{dx}=6\sqrt[3]{x}\,(3x+5x^4)^5(3+20x^3)+\frac{(3x+5x^4)^6}{3\sqrt[3]{x^2}}}$$

TRIG LOG DIFFERENTION ARCTRIG EQUATIONS OF TANGENT & NORMAL

DERIVATIVES

$\sin'(x) = \cos(x)$

$\cos'(x) = -\sin(x)$

$\tan'(x) = \sec^2(x)$

$\sec'(x) = \sec(x)\tan(x)$

$\csc'(x) = -\csc(x)\cot(x)$

$\cot'(x) = -\csc^2(x)$

$\arcsin'(x) = \frac{1}{\sqrt{1-x^2}}$

$\arccos'(x) = -\frac{1}{\sqrt{1-x^2}}$

$\arctan'(x) = \frac{1}{1+x^2}$

$\ln'(x) = \frac{1}{x}$

$\log_a'(x) = \frac{1}{x\ln a}$

$\boxed{\text{SIN}}$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h\to 0} \frac{\sin x\cosh + \sin h\cos x - \sin x}{h}$$

$$\lim_{h\to 0} \cancel{\frac{\sin h}{h}} \cdot \cos x + \sin x \cdot \frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}$$

$$\lim_{h\to 0} \cos x + \sin x \cdot \frac{\cos^2 h - 1}{h(\cos h + 1)}$$

$$\lim_{h\to 0} \cos x + \sin x \cdot \cancel{\frac{\sin h}{h}} \cdot \frac{-\sin h}{\cos h + 1}$$

$$\cos x + \sin x \cdot \frac{-\sin 0}{\cos(0) + 1}$$

$$\cos x + \sin x \cdot \frac{0}{2}$$

$$\boxed{\cos x}$$

$\boxed{\text{ARCSEC}}$

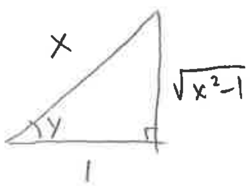
$y = \text{arcsec}\, x$

$x = \sec y$

$1 = \sec y \tan y \frac{dy}{dx}$

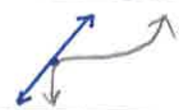
$1 = (x)(\sqrt{x^2-1}) \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$$



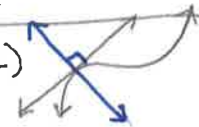
$\boxed{\text{Equation of a Tangent line of } f(x) \text{ at point } x = c}$

$$y - f(c) = f'(c)(x - c)$$



$\boxed{\text{Equation of a Normal line of } f(x) \text{ at point } x = c}$

$$y - f(c) = -\frac{1}{f'(c)}(x - c)$$



$\boxed{\text{sample}}$

$$f(x) = \frac{2+\sin t}{t+2}$$

$$\frac{df}{dt} = \frac{(t+2)(\cos t) - 1\cdot(2+\sin t)}{(t+2)^2}$$

$$\frac{df}{dt} = \frac{t\cos t + 2\cos t - 2 - \sin t}{(t+2)^2}$$

Process

1. Find x and y coordinates
2. Find $\frac{df}{dx}$
3. Calculate $\frac{df}{dx}\big|_{x=1}$
4. put into $y - y_1 = m(x - x_1)$

tangent line of $f(x) = x^3(2-3x^2)$ at $x = 1$

1. $x = 1 \quad y = 1^3(2-3)^2 = 1$
2. $\frac{df}{dx} = 3x^2(2-3x^2) + x^3(2)(3)(2-3x)$
3. $\frac{df}{dx}\big|_{x=1} = (3)(2-3)^2 + (1^3)\cdot(2)\cdot(-3)(2-3)$
$= 3 + 6 = 9$
4. $y - 1 = 9(x-1)$

The tangent line

Logs differentiation:

$\ln'(x) = \frac{1}{x}$

$\log_a'(x) = \frac{1}{x \ln a}$

sample. $y = \frac{e^{2x}\sqrt{x-3x^2}}{(\sin x)(5x^2+x)^7}$.

$$\ln y = \ln\left(\frac{e^{2x}\sqrt{x-3x^2}}{(\sin x)(5x^2+x)^7}\right)$$

log calc rule $\Rightarrow$ $\ln y = \ln e^{2x} + \ln\sqrt{x-3x^2} - \ln \sin x - \ln(5x^2+x)^7$

differentiate $\Rightarrow$ $\frac{dy}{dx} = 2x \cdot 1 + \frac{1}{2} \cdot \frac{1-6x}{x-3x^2} - \frac{\cos x}{\sin x} - 7 \cdot \frac{10x+1}{5x^2+x}$

Trig and normal line Example

Find the equation of the normal line $x = \frac{\pi}{3}$ for $y = x^2 \tan(x)$

$y' = 2x \tan x + x^2 \sec^2 x$

$x = \frac{\pi}{3}$

$y = \left(\frac{\pi}{3}\right)^2 \tan\left(\frac{\pi}{3}\right) = \frac{\pi^2\sqrt{3}}{9}$

$y - y_1 = \frac{1}{f'(c)}(x - x_1)$

$y - \frac{\pi^2\sqrt{3}}{9} = \frac{1}{2\left(\frac{\pi}{3}\right)\tan\frac{\pi}{3} + \left(\frac{\pi}{3}\right)^2 \sec^2\frac{\pi}{3}}\left(x - \frac{\pi}{3}\right)$

$y - \frac{\pi^2\sqrt{3}}{9} = \frac{1}{\frac{2\pi\sqrt{3}}{3}\left(\frac{3}{3}\right) + 4 \cdot \left(\frac{\pi}{3}\right)^2}\left(x - \frac{\pi}{3}\right)$

$y - \frac{\pi^2\sqrt{3}}{9} = \frac{1}{\frac{6\pi\sqrt{3}}{9} + \frac{4\pi^2}{9}}\left(x - \frac{\pi}{3}\right)$

$y - \frac{\pi^2\sqrt{3}}{9} = \frac{9}{6\pi\sqrt{3} + 4\pi^2}\left(x - \frac{\pi}{3}\right)$

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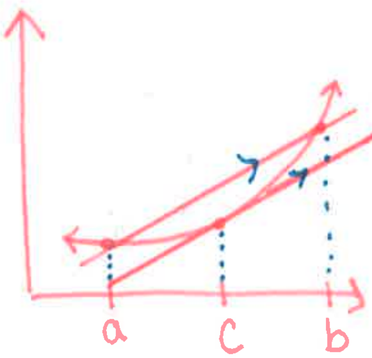
Group #5 → MVT, Rolle's Theorem, Kinematics

MVT = Mean Value Theorem

~ Suppose A function $f: D \to R$ is continuous on the closed interval $[a,b]$, & differentiable at the open interval $]a,b[$ then →

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$ for some number $c \in]a,b[$.



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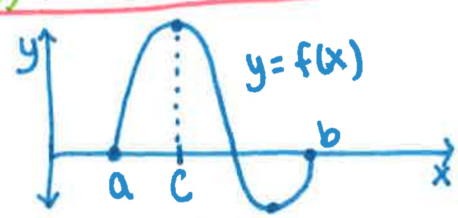
Group 5

Rolle's Theorem

~ Suppose A function $f: D \to R$ is continuous on the closed interval $[a,b]$, & differentiable at the open interval $]a,b[$.

~ IF $f(a) = f(b) = 0$, then there exists a value $c \in]a,b[$ such that $f'(c) = 0$



Kinematics:

~ Distance = $S(t)$ = The position of the object on the line is a function of time, t

~ Velocity = $v(t) = \frac{ds}{dt} = \lim_{h \to 0} \frac{S(t+h) - S(t)}{h}$ = The rate of change of $S(t)$ with respect to time

- Derivative of Distance
- Average velocity = $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$ → Speed: $|v(t)|$

~ Acceleration: $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \lim_{h \to 0} \frac{v(t+h) - v(t)}{h}$

- The rate of change of $v(t)$ with respect to time

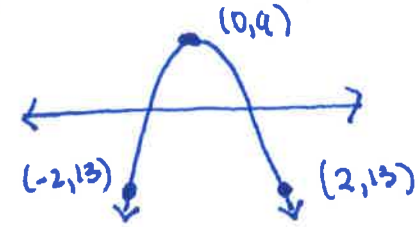
Notes: - The speed increase where signs of velocity & acceleration are same
- The speed decrease where signs of velocity & acceleration are opposite.

#1. Given $f(x) = \begin{cases} ax - 7 & x \geq 4 \\ x^2 - b & x < 4 \end{cases}$ $a = 8$ $b = -9$ show $\frac{df}{dx} = 0$ exists using Rolle's Theorem

$-2 < x < 2$

$f(x) = 8x - 7$ when $x \geq 4$ $\quad f(-2) = 13$ $\quad f(0) = 9$

$f(x) = x^2 + 9$ when $x < 4$ $\quad f(2) = 13$



$\therefore f'(c) = 0$ between $x = -2$ and $x = 2$

$\boxed{\frac{df}{dx} = 0 \text{ must exist}}$

#2. Find all numbers c in the interval $[0, 27]$ that satisfy the MVT for $f(x) = 6\sqrt[3]{x}$.

MVT $= f'(c) = \frac{f(b) - f(a)}{b - a}$

$f(27) = 6\sqrt[3]{27}$
$= 6 \cdot 3$
$= 18$

$= \frac{f(27) - f(0)}{27 - 0}$

$f(0) = 6\sqrt[3]{0}$
$= 0$

$= \frac{18 - 0}{27 - 0}$

$f'(c) = \left(\frac{1}{3}\right)(6)\left(x^{-\frac{2}{3}}\right)$

$= \frac{2}{3}$

$= \frac{2}{\sqrt[3]{x^2}}$

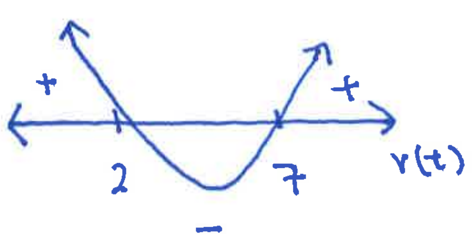
$\frac{2}{3} = \frac{2}{\sqrt[3]{x^2}}$

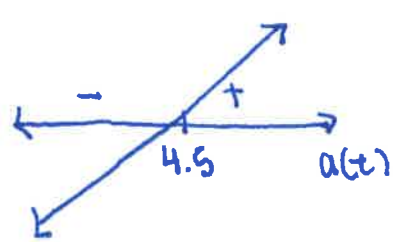
$3 = \sqrt[3]{x^2} \implies x = \pm 3\sqrt{3}$

c cannot be negative, $\therefore$ $\boxed{c = 3\sqrt{3}}$

#3. A particle's position from linear movement is $s(t) = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 14t + 2$, $t \geq 0$. On what intervals of time is the particle's speed increasing? Use sign diagrams.

$s'(t)$ or $v(t) = t^2 - 9t + 14$ $\quad s'(t) = (t - 7)(t - 2)$ $\quad s''(t) = 2t - 9$ or $a(t)$





$\boxed{\therefore t \in (7, \infty) \text{ when speed is increasing}}$

Curve Analysis

First Derivative Test	Second Derivative Test
the critical point $(c, f(c))$ is a relative local max where $f'(c)=0$ $f'(c)=0$ $f'(x)>0$ on left $f'(x)<0$ on right sign diagram: $f'(x)$: $+$ $x=c$ $-$	if $f''(c)<0$ at critical point $x=c$, $(c, f(c))$ is a relative max. $f''(c)<0$ concave down sign diagram: $f''(x)$: $-$ $x=c$
the critical point $(c, f(c))$ is a relative local min, where $f'(c)=0$ $f'(x)<0$ on left $f'(x)>0$ on right $f'(c)=0$ sign diagram: $f'(x)$: $-$ $x=c$ $+$	if $f''(c)>0$ at critical point $x=c$, $(c, f(c))$ is a relative min concave up $f''(c)>0$ sign diagram: $f''(x)$: $x=c$ $+$
the critical point $(c, f(c))$ is neither a relative local max, or a relative min, where $f'(c)=0$ $f'(x)>0$ on left $x=c$ $f'(x)>0$ on right $x=c$ $f'(x)<0$ on left $f'(x)<0$ on right sign diagram: $f'(x)$: $\pm$ or $\pm$ $x=c$	if $f''(d)=0$, $(d, f(d))$ is an inflection point. $x=d$ inflection point $x=d$ $f''(d)$ is undefined $x=d$ inflection point sign diagram: $f''(x)$: $\pm$ or $\mp$ $x=d$

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Group #4

1. $f(x) = 3x^4 - 8x^3 + 2$

$\frac{dy}{dx} = 12x^3 - 24x^2$ ← 1st derivative test

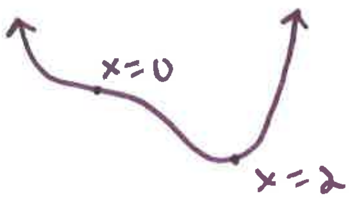
$0 = 12x^2(x-2)$

$x=0 \quad x=2$

sign of $f'(x)$

$-$ | $-$ | $+$ (at 0, 2)

$f'(x) = 12x^2(x-2)$



1) Increasing $(2, \infty)$
Decreasing $(-\infty, 0) \cup (0, 2)$

2) Local Max: None
Local Min: $x=2$

2. $g(x) = -x^3 + 3x^2 + 5$

$\frac{dy}{dx} = -3x^2 + 6x$

2nd derivative test

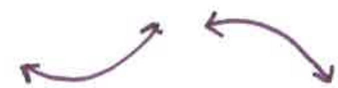
$\frac{d^2y}{dx^2} = -6x + 6 = 0$

$x = 1$

sign diagram of $g''(x)$

$g''(x) = -6(x-1)$

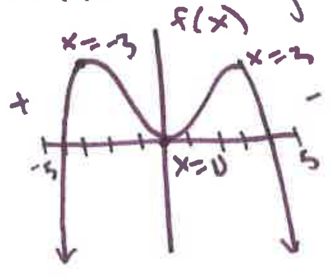
$x=0$: $+$ | $x=2$: $-$ (at 1)

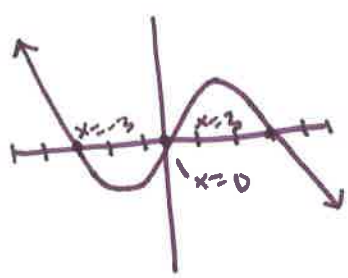


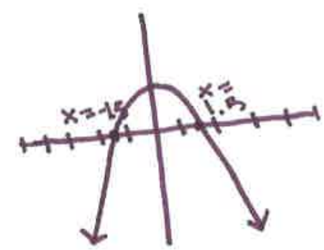
1) Inflection point:
$x=1$

2) Concave Up $(-\infty, 1)$
Concave Down $(1, \infty)$

3. Given the graph of $f(x)$, sketch the graphs of $f'(x)$ and $f''(x)$







4. Curve Analysis practice

1) $f(x) = \sin x + \cos x \quad x \in [0, 2\pi]$

a. Find the stationary points expressed in ordered pair (x,y)

$f'(x) = \cos x - \sin x$

$0 = \cos x - \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$(\frac{\pi}{4}, 0) \quad (\frac{5\pi}{4}, 0)$

b. Find the intervals where $f(x)$ is increasing and decreasing

$+$ | $-$ | $+$ (at $\frac{\pi}{4}$, $\frac{5\pi}{4}$)

Increasing: $(-\infty, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, \infty)$

Decreasing: $(\frac{\pi}{4}, \frac{5\pi}{4})$

c. Find the inflection points just for x values

$f''(x) = -\sin x - \cos x$

$0 = -1(\sin x + \cos x)$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

d. Find the intervals where $f(x)$ is concave up and down

$f''(x) = -\sin x - \cos x$

$-$ | $+$ | $-$ (at $\frac{\pi}{4}$, $\frac{5\pi}{4}$)

concave up: $(\frac{\pi}{4}, \frac{5\pi}{4})$

concave down: $(-\infty, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, \infty)$

e. State the Local max and min

Local Max $x = \frac{\pi}{4}$

Local Min $x = \frac{5\pi}{4}$

Optimization & Related Rate

Strategy

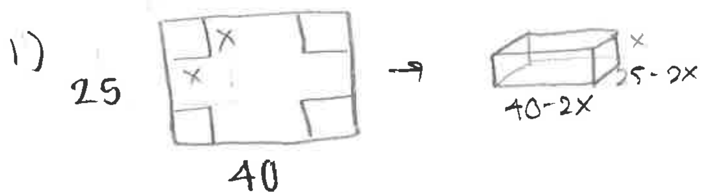
1. Draw a diagram of given situation with appropriate notation
2. Construct a formula with the variabe to be optimized
3. Find the first derivative & solve for x which makes the first derivative 0
4. Confirm if the solution is maximum or minimum by the 2nd derivative test.

$\frac{dy}{dx}$ gives the rate of change in y with respect to x

Examples

1. A rectangular cake dish is made by cutting out squares from the corners of a 25 cm x 40 cm rectangle, and then folding it to form the container. What size square must be cut out to produce the cake dish of max volume?

1)



2) volume

$(40-2x)(25-2x)(x)$

3) $\frac{dv}{dx}=0$

$V = 4x^3 - 130x^2 + 1000x$

$\frac{dv}{dx} = 12x^3 - 260x^2 + 1000x$

$= 4(x-5)(3x-50)=0$

4) $\frac{dv^2}{dx^2} = 24x - 200$

$\boxed{x=5 \text{ max}}$

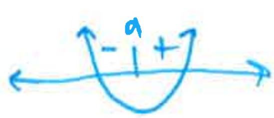
$\left.\frac{dv^2}{dx^2}\right|_{x=5}$ $24(5) - 260 < 0$ max

$\left.\frac{dv^2}{dx^2}\right|_{x=\frac{50}{3}}$ > 0 minimum

Testing optimization

Sign Diagram

local max



local min

Second Derivative

$\frac{d^2y}{dx^2} < 0 =$ local max

$\frac{d^2y}{dx^2} > 0 =$ local min

Graphical

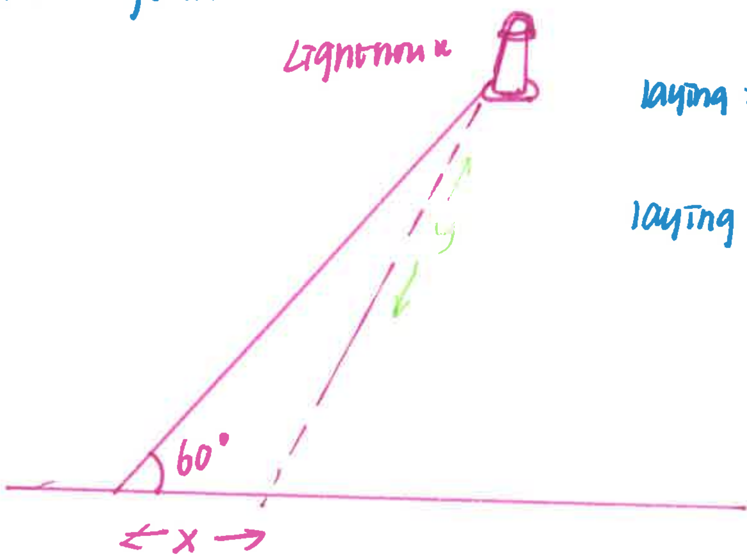
graph of f(x)

local max

local min

IB Question

An electricity station is on the edge of a straight coastline. A Light house is located in the sea 200m from the electricity station. The angle between the coastline and the line joining the lighthouse with the electricity station is 60°. A cable needs to be laid connecting the lighthouse to the electricity station. It is decided to lay the cable in a straight line to the coast and then along the coast to the electricity station. The length of cable laid along the coastline is x metres. This information is illustrated in the diagram below:



laying the cable along the sea bed:
US$ 80 per metre

laying on land
US $ 20 per m

(a) in terms of x, expression for the cost

(b) Find value of x, this cost is minimized

Chapter 14: Vectors

(2D) Vectors:

basic: $\vec{a} = a_1 i + a_2 j$

column: $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} i \\ j \\ k \end{pmatrix}$

magnitude: $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$ OR $\sqrt{i^2 + j^2}$

unit vector: $\dfrac{\vec{a}}{|\vec{a}|} = \dfrac{a_1 i + a_2 j}{\sqrt{a_1^2 + a_2^2}}$

(3D vectors):

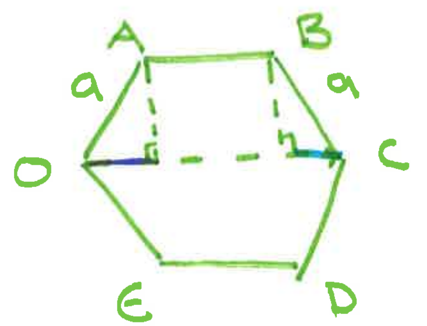
basic: $\vec{b} = b_1 i + b_2 j + b_3 k$

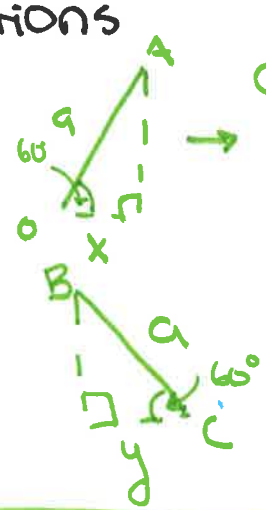
column: $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

magnitude: $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

unit vector: $\dfrac{\vec{b}}{|\vec{b}|} = \dfrac{b_1 i + b_2 j + b_3 k}{\sqrt{b_1^2 + b_2^2 + b_3^2}}$

ex: #4 from IB Questions





$\cos 60° = \dfrac{x}{a}$

$x = a \cdot \cos 60°$

$\cos 60° = \dfrac{y}{a}$

$y = a \cdot \cos 60°$

$\vec{OC} = a \cdot \cos 60° + AB + a \cdot \cos 60°$

$= 2a\cos 60° + a$

$= a + a = \boxed{2a}$

ex: Geometric Properties of the Cross Product WU

Given $A(1,4,2)$ & $B(0,8,0)$ & $C(4,10,3)$

1) Find $m\angle A\hat{B}C$ in radians

- $\vec{BA} = (1,-4,2)$ & $\vec{BC} = (4,2,3)$

- $\cos\theta = \dfrac{\vec{BA}\cdot\vec{BC}}{|BA|\cdot|BC|} = \dfrac{(1)(4)+(-4)(2)+(2)(3)}{\sqrt{1^2+(-4)^2+2^2}\cdot\sqrt{4^2+2^2+3^2}}$

$\cos\theta = \dfrac{4-8+6}{\sqrt{21}\cdot\sqrt{29}}$ $\boxed{\theta = 1.4897}$

2) ~~BA~~ Find the area of $\triangle ABC$

$\text{area} = \dfrac{|a||b|\cdot\sin\theta}{2} = \dfrac{\sqrt{21}\cdot\sqrt{29}\cdot\sin(1.4897)}{2}$

$= \boxed{14.3}$

3) Find vector v, which is perpendicular to $\vec{BA}$ & $\vec{BC}$

$\begin{vmatrix} i & j & k \\ 1 & -4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = \begin{matrix} i(-12-4) \\ -j(3-8) \\ k(2+16) \end{matrix} = \boxed{-16i+5j+18k}$

4) find $|v|$

$\sqrt{(-16)^2+5^2+(18)^2} = 28.6$

5) & the area?

$\dfrac{28.6}{2} = 14.3$

$\therefore \dfrac{|\vec{a}\times\vec{b}|}{2} = \dfrac{|a||b|\cdot\sin\theta}{2}$