IB HL 1 **Algebraic Approach** **Name:** ________________ **Period:** __________

Find limits algebraically.

1) Substitution Method:

$\lim_{x\to 2}(2x^3-3x^2+1) = 2(2)^3-3(2)^2+1$

$= \boxed{5}$

$\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{x-1} = \frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}-1} = \frac{1}{\frac{\pi}{2}-1} = \frac{1}{\frac{\pi}{2}-\frac{2}{2}}$

$= \boxed{\frac{2}{\pi-2}}$

2) Fractional reduction Method (when substitution method does not work)

$\lim_{x\to 2}\frac{x^2+x-6}{x-2} = \frac{(2)^2+2-6}{2-2} = \frac{0}{0}$

$= \lim_{x\to 2}\frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$

$= \lim_{x\to 2}(x+3) = 2+3 = \boxed{5}$

$\lim_{x\to 4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)}$

$= \lim_{x\to 4}\frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)}$

$= \lim_{x\to 4}\frac{1}{\sqrt{x}+2}$

$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

$\lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{1}{3}}{x}$

$= \lim_{x\to 0}\frac{\frac{3}{3(3+x)}-\frac{3+x}{3(3+x)}}{x}$

$= \lim_{x\to 0}\frac{\frac{\cancel{3}-\cancel{3}-x}{3(3+x)}}{\frac{x}{1}}$

$= \lim_{x\to 0}\frac{-\cancel{x}}{3\cancel{x}(3+x)}$

$= \lim_{x\to 0}\frac{-1}{3(3+x)} = \frac{-1}{9}$

Practice)

a. $\lim_{x\to 6}\frac{\sqrt{x+3}-3}{x-6}$

$= \lim_{x\to 6}\frac{(\sqrt{x+3}-3)(\sqrt{x+3}+3)}{(x-6)[\sqrt{x+3}+3]}$

$= \lim_{x\to 6}\frac{(x+3-9)}{(x-6)[\sqrt{x+3}+3]}$

$= \lim_{x\to 6}\frac{\cancel{(x-6)}}{\cancel{(x-6)}[\sqrt{x+3}+3)}$

$= \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$

b. $\lim_{x\to\frac{1}{3}}\frac{\frac{1}{x}-3}{3x-1}$

$= \lim_{x\to\frac{1}{3}}\frac{\left(\frac{1}{x}-3\right)\cdot x}{(3x-1)\cdot x}$

$= \lim_{x\to\frac{1}{3}}\frac{\cancel{(1-3x)}\ -1}{\cancel{(3x-1)}\cdot x}$

$= \lim_{x\to\frac{1}{3}}\frac{-1}{x}$

$= \boxed{-3}$

c. $\lim_{x\to\frac{\pi}{4}}\frac{\cos^2 x-\sin^2 x}{\cos x-\sin x}$

$= \lim_{x\to\frac{\pi}{4}}\frac{(\cos x+\sin x)\cancel{(\cos x-\sin x)}}{\cancel{(\cos x-\sin x)}}$

$= \lim_{x\to\frac{\pi}{4}}\cos x+\sin x$

$= \left(\cos\frac{\pi}{4}\right)+\left(\sin\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}$

$= \boxed{\sqrt{2}}$