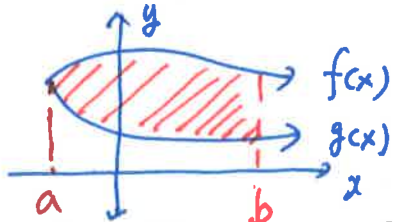
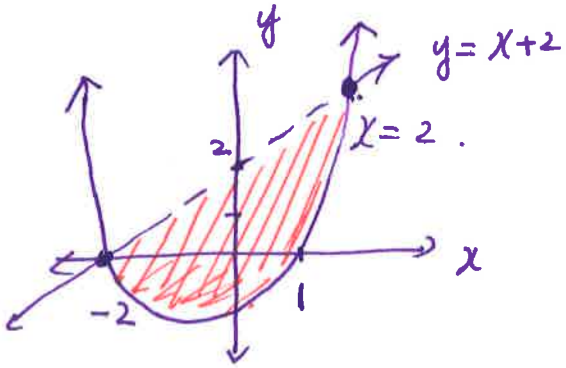
22B Area between Curves

$\Rightarrow$



$\Rightarrow \int_a^b [f(x)-g(x)]\,dx$

1. Find the area of the region enclosed by $y=x+2$ and $y=x^2+x-2$.



$y=(x+2)(x-1)$

$x=-2 \quad x=1$

$\cancel{x}+2 = x^2+\cancel{x}-2$

$0 = x^2-4 \qquad \Rightarrow x=2$

$0=(x-2)(x+2) \qquad x=-2$

$\frac{16}{3} \quad \begin{matrix}48\\-16\end{matrix} \qquad 8+8-\frac{8}{3}-\frac{8}{3}$

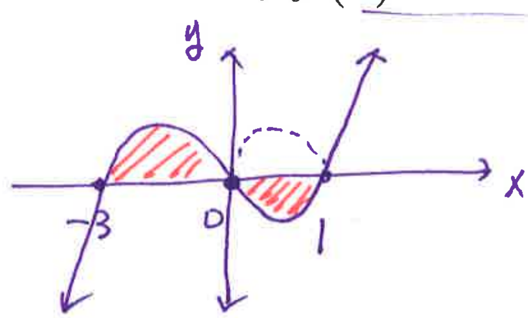
$16-\frac{16}{3}$

$$\text{Area} = \int_{-2}^{2} \left[(x+2)-(x^2+x-2)\right]dx$$

$$= \int_{-2}^{2}(-x^2+4)\,dx = \left[-\frac{1}{3}x^3+4x\right]_{x=-2}^{x=2} = \left[-\frac{1}{3}(2)^3+4(2)\right]-\left[-\frac{1}{3}(-2)^3+4(-2)\right]$$

$$= \boxed{\frac{32}{3}}$$

2. Find the total area of the regions contained by $f(x)=x^3+2x^2-3x$ and the *x*-axis.



$f(x) = x^3+2x^2-3x=0$

$x(x^2+2x-3)=0$

$x(x+3)(x-1)=0$

$x=0, \; x=-3, \; x=1$

$$\text{Area} \Rightarrow \int_{-3}^{1} |x^3+2x^2-3x|\,dx$$

$$= \int_{-3}^{0}(x^3+2x^2-3x)\,dx - \int_{0}^{1}(x^3+2x^2-3x)\,dx$$

$$= \left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{x=-3}^{x=0} - \left[\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2\right]_{x=0}^{x=1}$$

$$= \frac{71}{6} = 11.8\overline{3}$$

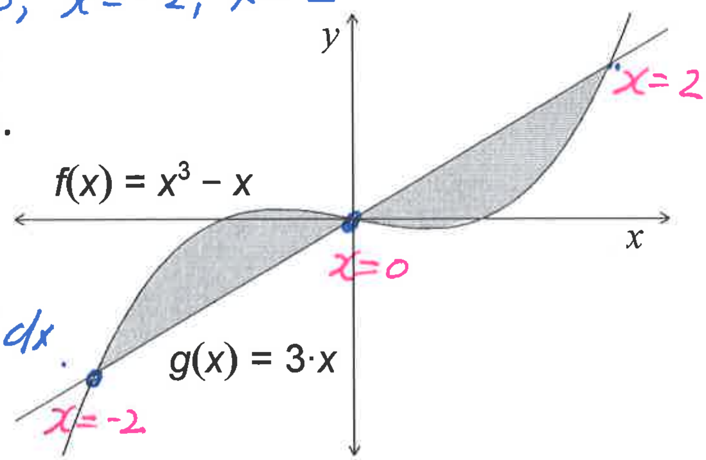
3. Find the area between $f(x)=x^3-x$ and $g(x)=3x$.

a. Use your GFC to find the points of intersection of f and g.

$$x^3-x=3x \Rightarrow x^3-4x=0 \Rightarrow x(x^2-4)=0$$

$$x(x+2)(x-2)=0 \Rightarrow x=0,\ x=-2,\ x=2$$

b. Represent the total shaded area using two integrals and show work to evaluate.



Total Area

$$= \int_{-2}^{0}\left[(x^3-x)-(3x)\right]dx + \int_{0}^{2}\left(3x-(x^3-x)\right)dx$$

$$= \int_{-2}^{0}\left[x^3-4x\right]dx + \int_{0}^{2}\left[-x^3+4x\right]dx$$

$$= \left[\frac{x^4}{4}-2x^2\right]_{x=-2}^{0} + \left[-\frac{x^4}{4}+2x^2\right]_{x=0}^{x=2}$$

$$= 0-\left[\frac{16}{4}-8\right]+\left[-\frac{16}{4}+8\right]-0$$

$$= 8-4-4+8=\boxed{8}$$

Area between Curves

$$\Rightarrow \int_a^b \left(f(x)-g(x)\right)dx$$

upper curve　　Low curve

$$= \int_a^b \left|f(x)-g(x)\right|dx$$

c. Represent the total area using one integral.

$$\text{Total Area} = \int_{-2}^{2}\left|(x^3-x)-3x\right|dx = \int_{-2}^{2}\left|x^3-4x\right|dx$$

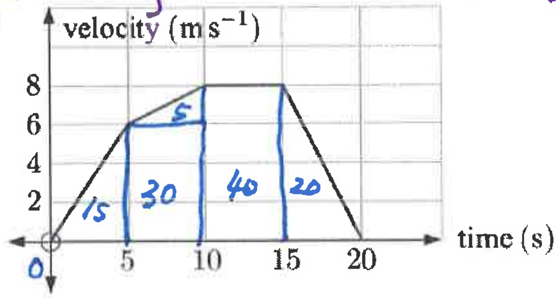
d. Evaluate this integral using your GFC.

$$v(t)=\frac{ds(t)}{dt} \Rightarrow \int ds(t)=\int v(t)\,dt \Rightarrow s(t)=\int v(t)\,dt$$

22C Kinematics

$$a(t)=\frac{dv(t)}{dt} \Rightarrow \int dv(t)=\int a(t)\,dt \Rightarrow v(t)=\int a(t)\,dt.$$

1. A runner has the velocity-time graph shown. Find the total distance traveled by the runner.

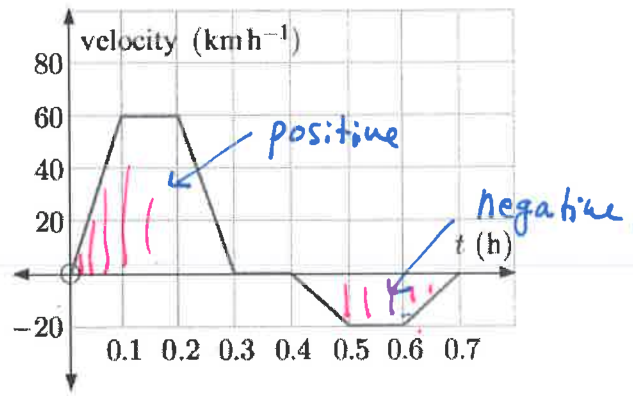


$$\text{Total distance} = \int_0^{20} v(t)\,dt$$

$$= 15+5+30+40+20$$

$$= \boxed{110\text{ m}}$$

2. A car travels along a straight road with the velocity-time function, $v(t)$, illustrated.



a. What is the significance of the graph:
i. above the t-axis?

$\Rightarrow$ Moving in the positive direction

ii. below the t-axis?

$\Rightarrow$ Moving in the negative direction.

b. Find the total distance travelled by the car.

$$\text{Total distance} = \int_{t_1}^{t_2} |v(t)|\,dt = \left(\tfrac{1}{2}\right)(60)(0.3+0.1) + \tfrac{1}{2}(20)(0.3+0.1)$$

$$= 12 + 4 = \boxed{16\text{ km}}$$

c. Find the final displacement of the car from its starting point.

Net displacement.

$$\text{Final displacement} = \int_{t_1}^{t_2} v(t)\,dt = 12-4 = \boxed{8\text{ km}}$$

d. Represent parts b and c using integrals.

$$\Rightarrow \text{Total Distance} = \int_0^{0.7} |v(t)|\,dt$$

$$\text{Final Displacement} = \int_0^{0.7} v(t)\,dt.$$

Using differential Calculus				**Using integral Calculus**
Position function $s(t)$	Velocity function $v(t) = \frac{ds}{dt}$	Acceleration function $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	Speed $\lvert v(t) \rvert$	Displacement (Net) $D = \int_{t_1}^{t_2} v(t)\,dt = S(t_2) - S(t_1)$
Total Distance Traveled				Total Distance Traveled $T = \int_{t_1}^{t_2} \lvert v(t) \rvert\, dt.$

3. A particle P moves in a straight line with velocity function $v(t) = t^2 - 3t + 2$ m/sec, $t \geq 0$.

a. If the particle's initial position is at 7, write an equation for the particle's position at any time $t \geq 0$.

$t = 0 \Rightarrow S = 7$

$$S(t) = \int v(t)dt = \int (t^2 - 3t + 2)dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + C$$

$C = 7$

$$\Rightarrow \boxed{S(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + 7}$$

b. Find the displacement of P after 4 seconds.

$$\text{Displacement} = \int_0^4 (t^2 - 3t + 2)dt = S(4) - S(0)$$

$$= \left[\frac{1}{3}(4)^3 - \frac{3}{2}(4)^2 + 2(4) + 7\right] - [7]$$

$$= \boxed{\frac{16}{3} \text{ m}}$$

c. How far does P travel in the first 4 seconds of motion?

Total distance

$$S(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + 7$$

$$\text{Total distance} = \int_0^4 \lvert t^2 - 3t + 2 \rvert dt = \lvert S(1) - S(0) \rvert + \lvert S(2) - S(1) \rvert + \lvert S(4) - S(2) \rvert$$

v(t)

$$= \lvert 7.8\overline{3} - 7 \rvert + \lvert 7.\overline{6} - 7.8\overline{3} \rvert + \lvert 12.\overline{3} - 7.\overline{6} \rvert$$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1 \quad t = 2$

0 ⊕ 1 ⊖ 2 ⊕ 4

$$= \boxed{5.\overline{6}} = \boxed{\frac{17}{3} \text{ m}}$$