22A Area under between two Curves Day two
(Horizontal Representative rectangles)

Area between two curves.

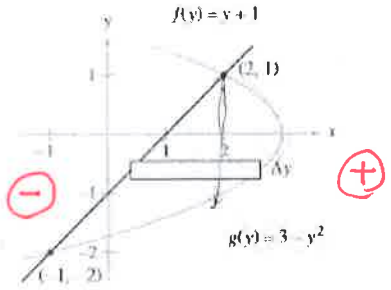
$$\lim_{n\to\infty}\sum_{i=1}^{n}[f(y_i)-g(y_i)]\Delta y =$$

$h = f(y) - g(y)$

↑ Right curve ↑ Left curve

$$\int_{y_1}^{y_2}\left(f(y)-g(y)\right)dy \quad \text{(FTC)}$$

$$\Delta y = \frac{y_2 - y_1}{n}$$

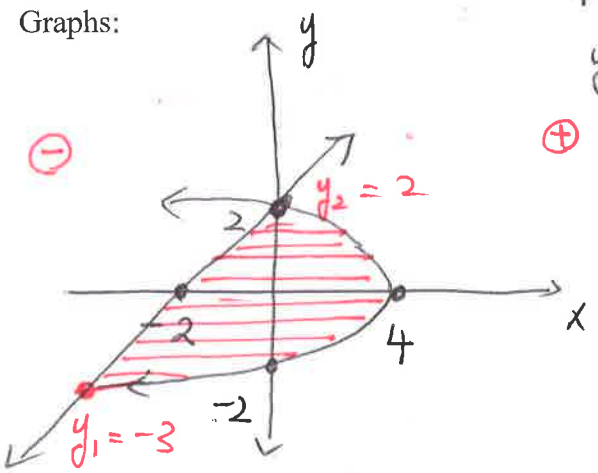


Horizontal rectangles (integration with respect to y)

Example 1) Find the area of the region bounded by the graphs of $x = 4 - y^2$ and $x = y - 2$.

Solution:

Graphs:



$4 - y^2 = y - 2$

$y^2 + y - 6 = 0$

$(y+3)(y-2) = 0$

$y = -3 \quad y = 2$

$$\Rightarrow \int_{-3}^{2}[(4-y^2)-(y-2)]\,dy$$

$$= \int_{-3}^{2}(-y^2 - y + 6)\,dy$$

$$= \left[-\frac{1}{3}y^3 - \frac{1}{2}y^2 + 6y\right]_{-3}^{2}$$

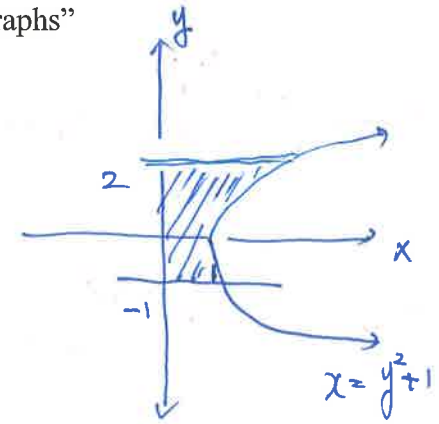
$$= \left[-\frac{8}{3} - 2 + 12\right] - \left[9 - \frac{9}{2} - 18\right]$$

$$= 30 - 11 + \frac{9}{2} - \frac{8}{3} = \frac{125}{6}$$

Practice) Find the area of the region bounded by the graphs of $f(y) = y^2 + 1$, $f(y) = 0$, $y = -1$, and $y = 2$.

Solution:

Graphs"



$$\int_{-1}^{2}(y^2+1)\,dy$$

$$= \left[\frac{1}{3}y^3 + y\right]_{-1}^{2} = \left[\frac{4}{3} + 2\right] - \left[\frac{1}{3} - 1\right] = 6$$